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Comprehensive Work on Interval-valued Fuzzy Translation and Multiplication in Z-subalgebra of Z-algebra

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In this paper, we explore interval-valued fuzzy translation and multiplication in z-algebras, delving into their detailed properties through the application of the concepts of fuzzy z-algebras and fuzzy z-subalgebras in z-algebras. Our specific focus is on presenting the notions of int-val-fuz-translation and multiplication, along with elucidating their homomorphism properties within the scope of this work. Additionally, we investigate several results based on in-val-fuz magnified translation in z-subalgebras.

Keywords: fuzzy set, fuzzy z-algebra, fuzzy z-subalgebra, interval-valued fuzzy multiplication, interval-valued fuzzy set, interval-valued fuzzy z-subalgebra, interval-valued fuzzy translation

INTRODUCTION

Zadeh (1965) introduced the idea of fuzzy sets. Several researchers looked at the theme of fuzzy subsets in general. The analysis of fuzzy subsets and their implementations in different analytical contexts gave rise to what is now known as fuzzy mathematics. The study by Aub Aynb Ansari and Chandramouleeswaran (2014) explores how fuzzy translations are applied to fuzzy β-subalgebras within the context of β-algebras. In this way, Barbiya (2015) discussed fuzzy multiplication and fuzzy translation for B-algebras, as well as fuzzy extensions of BG-ideals on BG-algebras, and few of its properties are established. Also, introduced the characteristics of fuzzy translation and multiplication in BRK-algebra. After that, BRK-subalgebras and BRK-ideals were examined and talked about in relation to fuzzy translation and fuzzy multiplication. Finally, on BRK-algebra, presented the idea of fuzzy magnified αβ-translation by Alsheri (2021). Hemavathi *et.al.* (2016) described fuzzy extensions, fuzzy α-translation, and fuzzy β-multiplications of fuzzy z-subalgebras (fuzzy z-ideals) of z-algebras, and some important features are also demonstrated. The thought of z-algebras has been initiated by the author in Chandramouleeswaran (2017). Sujatha and Muralikrishna (2015) introduced the notation of an intuitionistic fuzzy α-translation – in particular, $\alpha \in [0,1]$ to β-subalgebras of a β-algebra – and a few of their interesting and straightforward results were evaluated. The concept of bipolar fuzzy translation in BCK/BCI-algebras was discussed by Jun and Kim (2009).

The MBJ-neutrosophic magnified translation (MBJNMT) on G-algebra, a kind of formation of translation and multiplication, looked at the important

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outcomes of MBJ-neutrosophic ideal and MBJ-neutrosophic subalgebra utilizing the MBJNMT principle by Khalid and Jun (2020). After assessing how the MBJ-neutrosophic ideal and MBJ-neutrosophic subalgebra converted to an additional one, some significant MBJ-neutrosophic magnified translation results were developed using the intersection and union theory. The work of Khalid (2019) discussed the phenomena of ideals over G-algebra and intuitionistic fuzzy translation to intuitionistic fuzzy subalgebra, as well as some associated features. It discusses the concepts of G-algebra ideals, intuitionistic fuzzy extension, and intuitionistic fuzzy multiplication of intuitionistic fuzzy subalgebra. Numerous studies were also conducted between intuitionistic fuzzy ideals and subalgebra in G-algebra, as well as intuitionistic fuzzy extension, intuitionistic fuzzy translation, and intuitionistic fuzzy multiplication.

The concept of fuzzy translation and fuzzy multiplication was first introduced by Priya and Ramachandran (2014) using the fundamental idea of PS-algebra and their characteristics were also examined. The notation of fuzzy translation of a fuzzy set in UP-algebras was used to analyze some fascinating results of Iampan (2017). Algebraic characteristics of w-fuzzy translation and multiplication in BH-algebras were addressed by Prasana and Preamkumar (2020). Additionally, Prasana *et al.* (2021) studied fuzzy translation and multiplication in BG-algebras. The concepts of fuzzy translation and fuzzy multiplication of z-algebras, as well as z-homomorphism and cartesian product on fuzzy translation and fuzzy multiplication of z-algebras, were introduced in Sowmiya and Jeyalakshmi (2020a, b). Jana *et al.* (2016, 2017, 2019) contributed more to their relevant research and investigated many intriguing properties. Here, we briefly discuss some results on interval-valued fuzzy translation and multiplication in z-algebras, as well as a few of their detailed features.

PRELIMINARIES

The fundamental definitions are discussed in this section that are necessary for this research article.

Definition 2.1 (Aub Aynb Ansari and Chandramouleeswaran 2014; Zadeh 1965). Let G be a non-empty set. A fuzzy set S in G that is defined by a membership function γ is related to each point z in G, a real number $\gamma(z)$ in the interval [0, 1] representing the "grade of the membership" of G in S, *i.e.* a fuzzy set S belongs to G that is characterized by a membership function $\gamma: G \rightarrow [0, 1]$.

Definition 2.2 (Hemavathi *et al.* 2016; Jun and Kim 2009). An In-val-fuz set S described in G is represented as $S = \{(z, [\gamma^L(z), \gamma^U(z)])\}\ \forall\ z \in G$ (briefly denoted by $S = [\gamma^L, \gamma^U]$),

where γ^L and γ^U are two fuzzy sets in S such that $\gamma^L(z) \leq \gamma^U(z), \forall z \in G$.

Let $\bar{\gamma}(z) = [\gamma^L(z), \gamma^U(z)]\ \forall\ z \in G$. Consider the family of all closed subintervals of C [0, 1]. If $\gamma^L(z) = \gamma^U(z) =$ a, where $0 \leq a \leq 1$, then we have $\bar{\gamma}(z) = [a, a] = \bar{a}$. Thus, $\bar{\gamma}(z) \in C[0, 1]\ \forall\ z \in G$.

∴ The In-val-fuz set S follows, $S = \{(z, \bar{\gamma}(z))\}$, for all $z \in G$, where $\bar{\gamma}: G \rightarrow C[0, 1]$.

Let us define the refined minimum (abbreviated rmin) consisting of two elements in C[0, 1]. In addition, describe the characters "≥," "≤," and "=" when C[0, 1] contains two elements.

Consider, two elements $C_1=[a_1, b_1]$ and $C_2=[a_2, b_2] \in C[0,1]$.

Then, rmin $(C_1, C_2) = [\min\{a_1, a_2\}, \min\{b_1, b_2\}]$

$C_1 \geq C_2$ iff $a_1 \geq a_2, b_1 \geq b_2$.

Similarly, $C_1 \leq C_2$ and $C_1 = C_2$.

Definition 2.3 (Chandramouleeswaran 2017). Let G be a non-empty set that contains the binary operation * and a constant 0 is a Z-algebra (G,*,0) if the below properties hold:

1. $s * 0 = 0$;
2. $0 * s = s$;
3. $s * s = s$; and
4. $s * t = t * s$, when $s \neq 0$ and $t \neq 0$; $\forall s, t \in G$.

Example 2.4. Assuming the set G = {0, α, β, δ} contains the binary operation ∗ and a constant that is 0 specified on G with the Cayley table that follows is a z-algebra:

∗	0	α	β	δ
0	0	α	β	δ
α	0	α	0	α
β	0	0	β	β
δ	0	β	β	δ

Definition 2.5 (Sowmiya and Jayalakshmi 2020a). Consider (G,∗,0) to be a z-algebra. A fuzzy set S in G contains a membership function γ is called a fuzzy z-subalgebra of G if it satisfies the below condition:

$\gamma(u * v) \geq \min\{\gamma(u), \gamma(v)\}\ \forall\ u, v \in G.$

Definition 2.6 (Hemavathi *et al.* 2016; Sujatha and Muralikrishna 2015). Let (G, ∗, 0) be a z-algebra. An In-val-fuz set $\overline{\gamma}$ in G is said to be an In-val-fuz-z-subalgebra of a z-algebra in G if:

$\overline{\gamma}(u * v) \geq \mathrm{rmin}\{\overline{\gamma}(u), \overline{\gamma}(v)\}\ \forall\ u, v \in G.$

Example 2.7. If G = {0,1,2} be a set that contains this Cayley table:

∗	0	1	2	3
0	0	1	2	1
1	0	0	2	3
2	0	2	0	2
3	0	1	2	3

According to, an In-val-fuz set $\overline{\gamma}$ in G is:

$$\overline{\gamma}\,(z) = \begin{cases} [0.1, 0.4] & \text{if } z = 0,1 \\ [0.2, 0.6] & \text{if } z = 2 \\ [0.3, 0.5] & \text{if } z = 3 \end{cases}$$

Thus, $\overline{\gamma}$ be an In-val-fuz- z-sub algebra of G.

Definition 2.8 (Sowmiya and Jayalakshmi 2020b). Consider two z-algebras (J, ∗, 0) and (W, $*'$, $0'$). The map h:(J, ∗, 0) → $(W, *', 0')$ is then referred to as a z-homomorphism of z-algebras if $h(u * w) = h(u) *' h(w)$ ∀u, w ∈ J.

Definition 2.9 (Sowmiya and Jayalakshmi 2020b). Let h maps from J–W. Consider a fuzzy set R in G that contains a membership function γ_R. The pre-image (or inverse image) of T in h is then represented by h^{-1}(R) is the fuzzy set in J such as a membership function γ_h^{-1} (R) according to $\gamma_h^{-1}(R)\,(u) = \gamma_R(h(u))\ \forall\ u \in J$.

Remark 2.10 (Khalid 2019; Prasana *et al.* 2021). Let (G, ∗, 0) be a z-algebra and S of G is any fuzzy set, then it is expressed by Ŧ = 1 – sup {γ(u)/ u ∈ J} unless otherwise specified.

Definition 2.11 (Priya and Ramachandran 2014; Tanintorn and Supanat 2017). Let G be a non-empty set and S be a fuzzy set of a z-algebra G, and let φ ∈ [0, Ŧ]. A fuzzy φ-translation $\gamma_S^{Ŧ}$ of S, whose membership function $\gamma_{S_\varphi}^{Ŧ}$: G → [0, 1] is given by function $\gamma_{S_\varphi}^{Ŧ}(z) = \gamma(z) + \varphi$, ∀z ∈ G.

Definition 2.12 (Prasana and Preamkumar 2020). Let S be a fuzzy set of a z-algebra G and μ ∈ [0, 1]. A fuzzy μ-multiplication of S with a membership function $\gamma_{S_\mu}^{M}$: J → [0, 1] is characterized by $\gamma_{S_\mu}^{M}(z) = \mu.\ \gamma(z)$, ∀ z ∈ G.

Example 2.13. If G = {0, 1, 2} be a set that contains this Cayley table.

∗	0	1	2
0	0	1	2
1	0	0	2
2	0	2	0

Then, (G, $*$, 0) is a z-algebra.

Assume that a fuzzy set S in G with membership function as γ_s:

$$\gamma(z) = \begin{cases} 0.7 & \text{if } z = 0 \\ 0.3 & \text{if } z = 1 \\ 0.5 & \text{if } z = 1,2 \end{cases}$$

Let G be a fuzzy z-subalgebra. Here $Ŧ = 0.3$ and take $\varphi = 0.1 \in [0, Ŧ]$ and $\mu = 0.2 \in [0, 1]$. Then, the mapping $\gamma^{Ŧ}_{s_{0.1}}: G \rightarrow [0, 1]$ is a fuzzy 0.1-translation, and the mapping $\gamma^{M}_{s_{0.2}}: G \rightarrow [0, 1]$ is a fuzzy 0.2-multiplication.

Definition 2.14 (Jana and Pal 2017a, b). If h: $J \rightarrow W$ is a function, let $\bar{\gamma}_s$ and $\bar{\gamma}_R$ be the two In-val-fuz-$\bar{\varphi}$-translations onto J and W, respectively. Then, the inverse image of $\bar{\gamma}_R$ is based on h is represented by $h^{-1}(\bar{\gamma}_R) = \{ h^{-1}(\bar{\gamma}_R)\frac{Ŧ}{\varphi}(z): z \in G\}$ such that $h^{-1}(\bar{\gamma}_R)\frac{Ŧ}{\varphi}(z) = \bar{\gamma}_R(h(z) + \bar{\varphi})$.

Definition 2.15 (Sowmiya and Jayalakshmi 2020b). Let S and R be two fuzzy sets of G, each with membership functions γ_s and γ_R. Then, the Cartesian product $S \times R$ contains a membership function $\gamma_{S\times R}: G \times G \rightarrow [0, 1]$ is characterized by $\gamma_{S\times R}(z, w) \geq \text{rmin}\{\gamma_s(z), \gamma_R(w)\}, \forall z, w \in G$.

Definition 2.16 (Khalid 2019). Let $\bar{\gamma}$ represents an In-val-fuz subset of G, $\bar{\gamma} \in C[0, 1]$ and $\bar{\varphi} \in [\bar{0}, \bar{Ŧ}]$. The mapping $\bar{\gamma}^{NŦ}_{\bar{\varphi}\bar{\mu}}: G \rightarrow C[0, 1]$ is known as In-val-magnified fuzzy $\bar{\mu}\bar{\varphi}$ – translation of $\bar{\gamma}$ if it satisfies $\bar{\gamma}^{NŦ}_{\bar{\varphi}\tilde{\mu}}(z) = \bar{\mu}\bar{\gamma}(z) + \bar{\varphi}, \forall z \in G$.

INTERVAL-VALUED-FUZZY (IN-VAL FUZ) $\bar{\varphi}$-TRANSLATION OF Z-ALGEBRA

The notation of In-val-fuz-$\bar{\varphi}$-translation of z-algebra is discussed in this section. Let G be a z-algebra for any In-val-fuz set $\bar{\gamma}$ of G, which is denoted by $\bar{Ŧ} = \{[1, 1] - \text{rsup}\{\bar{\alpha}(z)/ z \in G\}\}$ unless stated otherwise.

Definition 3.1. Consider $\bar{\gamma}$ represents an In-val-fuz set of G and $\bar{\varphi} \in [\bar{0}, \bar{Ŧ}]$, where $\bar{\varphi} = [\varphi^L, \varphi^U]$ with $\varphi^L \in [0, Ŧ^L]$ and $\varphi^U \in [0, Ŧ^U]$ and $\bar{0} = [0, 0]$. The mapping $\bar{\gamma}^{\frac{Ŧ}{\varphi}}: G \rightarrow C[0, 1]$ is said that the In-val-fuz-$\bar{\varphi}$-translation of $\bar{\gamma}$ when it is satisfying $\bar{\gamma}^{\frac{Ŧ}{\varphi}}(z) = \bar{\gamma}(z) + \bar{\varphi}, \forall z \in G$.

Example 3.2. Consider the z-algebra $G = \{0, \alpha, \beta, \delta\}$ by using Example 2.4. Consider an In-val-fuz subset $\bar{\gamma}$ of G:

$$\bar{\gamma}(z) = \begin{cases} [0.3,0.5]: & z = 0 \\ [0.2,0.6]: & z = \alpha, \delta \\ [0.1,0.7]; & z = \beta \end{cases}$$

, then, let $\bar{\gamma}$ be an In-val-fuz- z-subalgebra of G.

Here $\bar{Ŧ} = \{[1, 1] - \text{rsup}\{\bar{\gamma}(z)/ z' \in Z\}\}$

$= \{[1, 1] - [0.3, 0.5]\}' = [0.5, 0.7]$.

Choose $\bar{\varphi} = [0.02, 0.04] \in [\bar{0}, \bar{Ŧ}]$.

Then, the In-val-fuz collection $\bar{\gamma}^{\frac{Ŧ}{\varphi}}: G \rightarrow C[0, 1]$ is given by $\bar{\gamma}^{\frac{Ŧ}{\varphi}}(0) = [0.52, 0.74]$, $\bar{\gamma}^{\frac{Ŧ}{\varphi}}(\alpha) = \bar{\gamma}^{\frac{Ŧ}{\varphi}}(\delta) = [0.42, 0.84]$, and let $\bar{\gamma}^{\frac{Ŧ}{\varphi}}(\beta) = [0.32, 0.94]$ be an In-val-fuz-$\bar{\varphi}$-translation of $\bar{\gamma}$.

Example 3.3. Consider the z-algebra $G = \{0, \alpha, \beta, \delta\}$ by using in Example 2.4. Consider an In-val-fuz subset $\bar{\gamma}$ of G by:

$$\bar{\gamma}(z) = \begin{cases} [0.1,0.4]: & z = 0 \\ [0.3,0.6]: & z = \alpha, \delta \\ [0.2,0.5]; & z = \beta \end{cases}$$

Then, $\bar{\gamma}$ be an In-val-fuz-z-subalgebra of G:

Here $\bar{Ŧ} = \{[1, 1] - \text{rsup}\{\bar{\gamma}(z)/ z' \in Z\}\}'$

$= \{[1, 1] - [0.1, 0.4]\} = [0.6, 0.9]$.

Choose $\bar{\varphi} = [0.4, 0.7] \in [\bar{0}, \bar{Ŧ}]$.

Then, the In-val-fuz collection $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}$: G → C[0, 1] is given by $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(0) = [1, 1.6]$,

$\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(\alpha) = \overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(\delta) = [0.8, 1.4]$, and $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(\beta) = [0.9, 1.5]$ is not an In-val-fuz-$\overline{\varphi}$-translation of $\overline{\gamma}$.

Theorem 3.4. For every In-val-fuz- z-subalgebra $\overline{\gamma}$ of G and $\overline{\varphi} \in [\overline{0}, \overline{Ŧ}]$, the In-val-fuz $\overline{\varphi}$-translation of $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}$ (z) of $\overline{\gamma}$ is an In-val-fuz-z-subalgebra of G.

Proof. Take u, v ∈ G and $\overline{\varphi} \in [\overline{0}, \overline{Ŧ}]$;

then, $\overline{\gamma}(u * v) \geq \text{rmin}\{\overline{\gamma}(u), \overline{\gamma}(v)\}$

Now, $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(u * v) = \overline{\gamma}(u * v) + \overline{\varphi}$

$\geq \text{rmin}\{\overline{\gamma}(u), \overline{\gamma}(v)\}\ \overline{\varphi}$

$= \text{rmin}\{\overline{\gamma}(u) + \overline{\varphi}, \overline{\gamma}(v) + \overline{\varphi}\}$

$= \text{rmin}\{\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(u), \overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(v)\}$

Thus, let $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}$ of $\overline{\gamma}$ be an In-val-fuz-z-subalgebra of G.

Now, using the converse of the previous theorems, it is as follows.

Theorem 3.5. For every In-val-fuz-subset $\overline{\gamma}$ of G and $\overline{\varphi} \in [\overline{0}, \overline{Ŧ}]$. If the In-val-fuz-$\overline{\varphi}$-translation of $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}$ (z) of $\overline{\gamma}$ is likewise an In-val-fuz- z-subalgebra of G, then is also an $\overline{\gamma}$.

Proof. Take u, v ∈ G.

Consider $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(z)$ of $\overline{\gamma}$ is an In-val-fuz-z-subalgebra of G for some $\overline{\varphi} \in [\overline{0}, \overline{Ŧ}]$.

Now, $\overline{\gamma}(u * v) + \overline{\varphi} = \overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(u * v)$

$\geq \text{rmin}\{\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(u), \overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(v)\}$

$= \text{rmin}\{\overline{\gamma}(u) + \overline{\varphi}, \overline{\gamma}(v) + \overline{\varphi}\}$

$= \text{rmin}\{\overline{\gamma}(u), \overline{\gamma}(v)\} + \overline{\varphi}$

$\Rightarrow \overline{\gamma}(u * v) \geq \text{rmin}\{\overline{\gamma}(u), \overline{\gamma}(v)\}$.

Hence, $\overline{\gamma}$ is the In-val-fuz- z-subalgebra of G.

Remark 3.6. For any In-val-fuzset $\overline{\gamma}$ of G, the In-val-fuz-$\overline{\varphi}$-translation of $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}$ ($\overline{\varphi} \in [\overline{0}, \overline{Ŧ}]$) of $\overline{\gamma}$ need not be an In-val-fuz- z-subalgebra of G.

If G is the z-algebra from Example 3.2, following the In-val-fuz set $\overline{\gamma}$:

$$\overline{\gamma}(z) = \begin{cases} [0.4, 0.6] & z = 0 \\ [0.1, 0.3] & z = \alpha \\ [0.2, 0.5] & z = \beta, \delta \end{cases}$$

Let $\overline{\varphi} = [0.02, 0.04]$. Then, the corresponding $\overline{\varphi}$-translation is:

$\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(0) = [0.42, 0.64]\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(\alpha) = [0.74, 0.92]$

$\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(\beta) = [0.54, 0.82]\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(\delta) = [0.54, 0.82]$

Now, $\overline{\gamma}(\alpha * \beta) = \overline{\gamma}(0) = [0.02, 0.15] \ngeq [0.4, 0.6]$

$= \text{rmin}\{\overline{\gamma}(\alpha), \overline{\gamma}(\beta)\}$, and

$\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(\alpha * \beta) = \overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(0) = [0.3996, 0.7544] \ngeq [0.42, 0.64] = \text{rmin}\{\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(\alpha), \overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}(\beta)\}$.

Hence, $\overline{\gamma}$ and $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}$ are not In-val-fuz z-subalgebra of G.

Corollary 3.7. Consider $\overline{\gamma}$ be an In-val-fuz set of G. If $\overline{\varphi} = \overline{0}$, then the In-val-fuz-$\overline{\varphi}$-translation $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}$ of $\overline{\gamma}$ be an In-val-fuz-z-subalgebra of G.

Theorem 3.8. Let $\overline{\gamma}$ be an In-val-fuz-z-subalgebra of G. Then, for $\overline{\varphi}, \overline{\varphi'} \in [\overline{0}, \overline{Ŧ}]$, $(\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}} \cap \overline{\gamma}\frac{\overline{Ŧ}}{\overline{\varphi'}})$ and $(\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}} \cup \overline{\gamma}\frac{\overline{Ŧ}}{\overline{\varphi'}})$ are also an In-val-fuz-z-subalgebra of G.

Proof. Assume that $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}$ and $\overline{\gamma}\frac{\overline{Ŧ}}{\overline{\varphi'}}$ are In-val-fuz-$\overline{\varphi}$-translation of an In-val-fuz- z-subalgebra of G, where $\overline{\varphi}$, $\overline{\varphi'} \in [\overline{0}, \overline{Ŧ}]$.

If $\overline{\varphi} \leq \overline{\varphi'}$, using Theorem 3.4, $\overline{\gamma}^{\overline{Ŧ}}_{\overline{\varphi}}$ and $\overline{\gamma}\frac{\overline{Ŧ}}{\overline{\varphi'}}$ are In-val-fuz-$\overline{\varphi}$-translation of z-subalgebra of G.

Now:

$(\overline{\gamma}^{\overline{\mathrm{T}}}_{\overline{\varphi}} \cap \overline{\gamma}\frac{\overline{\mathrm{T}}}{\overline{\varphi'}})$ (z) = rmin$\{\overline{\gamma}^{\overline{\mathrm{T}}}_{\overline{\varphi}}(z), \overline{\gamma}\frac{\overline{\mathrm{T}}}{\overline{\varphi'}}(z)\}$

= rmin$\{\overline{\gamma}(z) + \overline{\varphi}, \overline{\gamma}(z) + \overline{\varphi'}\}$

$= \overline{\gamma}(z) + \overline{\varphi}$

$= \overline{\gamma}^{\overline{\mathrm{T}}}_{\overline{\varphi}}(z)$

Also:

$(\overline{\gamma}^{\overline{\mathrm{T}}}_{\overline{\varphi}} \cup \overline{\gamma}\frac{\overline{\mathrm{T}}}{\overline{\varphi'}})$ (z) = rmax $(\overline{\gamma}^{\overline{\mathrm{T}}}_{\overline{\varphi}}(z), \overline{\gamma}\frac{\overline{\mathrm{T}}}{\overline{\varphi'}}(z)\}$

= rmax $\{\overline{\gamma}(z) + \overline{\varphi}, \overline{\gamma}(z) + \overline{\varphi'}\}$

$= \overline{\gamma}(z) + \overline{\varphi'}$

$= \overline{\gamma}\frac{\overline{\mathrm{T}}}{\overline{\varphi'}}(z)$

Hence, $(\overline{\gamma}^{\overline{\mathrm{T}}}_{\overline{\varphi}} \cap \overline{\gamma}\frac{\overline{\mathrm{T}}}{\overline{\varphi'}})$ and $(\overline{\gamma}^{\overline{\mathrm{T}}}_{\overline{\varphi}} \cup \overline{\gamma}\frac{\overline{\mathrm{T}}}{\overline{\varphi'}})$ are additionally an In-val-fuz- z-subalgebra of G.

Theorem 3.9. Assume that $\overline{\gamma}_1$ and $\overline{\gamma}_2$ are In-val-fuz-z-subalgebra of G. Let $\overline{\mathrm{T}}$ = rmin $\{\overline{\mathrm{T}}_{\overline{\gamma}_1}, \overline{\mathrm{T}}_{\overline{\gamma}_2}\}$, where $\overline{\mathrm{T}}_{\overline{\gamma}_1}$ = $\{[1, 1] - \text{rsup}\{\overline{\gamma}_1(z); z \in Z\}\}'$ and $\overline{\mathrm{T}}_{\overline{\gamma}_2}$ = $\{[1, 1] - \text{rsup}\{\overline{\gamma}_2(z); z \in Z\}\}'$. Then, the $\cap$ of $\overline{\varphi}$ – translation of $\overline{\gamma}_1$ and $\overline{\varphi'}$-translation of $\overline{\gamma}_2$ for some $\overline{\varphi}$, $\overline{\varphi'} \in [\overline{0}, \overline{\mathrm{T}}]$ is an In-val-fuz-z-subalgebra of G.

Proof. Consider $\overline{\gamma}_1$ and $\overline{\gamma}_2$ are In-val-fuz-z-subalgebra of G.

Then, by Theorem 3.4, $\overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}}$ and $\overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi'}}$ are In-val-fuz-z-subalgebra of G.

Take x, y $\in$ G.

Now:

$(\overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}} \cap \overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi'}})(x * y)$ = rmin$\{(x * y), \overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(x * y)\}\overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}}$

$\geq$ rmin {rmin{ $\overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(x), \overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(y),\}$, rmin$\{\overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(x), \overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi}}$ (y)}}

= rmin{rmin$\{\overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(x), \overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(x)\}$, rmin$\{\overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(y), \overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi}}$ (y)}}

= rmin$\{(\overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}} \cap \overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(x), (\overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}} \cap \overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(y))\}$

Hence, $(\overline{\gamma}_1\frac{\overline{\mathrm{T}}}{\overline{\varphi}} \cap \overline{\gamma}_2\frac{\overline{\mathrm{T}}}{\overline{\varphi'}})$ is an In-val-fuz-z-subalgebra of G.

Definition 3.10. If f: U → V is a function, let $\overline{\gamma}_u$ and $\overline{\gamma}_v$ be In-val-fuz-$\overline{\varphi}$-translation on U and V, respectively. Then, the inverse image of $\overline{\gamma}_v$ under g is defined by f^{-1} $(\overline{\gamma}_v) = \{f^{-1}(\overline{\gamma}_v)\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(z); z \in G\}$, such that $f^{-1}(\overline{\gamma}_v)\frac{\overline{\mathrm{T}}}{\overline{\varphi}}(z)$ $= \overline{\gamma}_v(f(z) + \overline{\varphi})$.

Theorem 3.11. Assuming U and V are two z-algebra, and f: U → V be a homomorphism. If the In-val-fuz-translation$\overline{\gamma}_V$ of V is a In-val-fuz z-subalgebra of V, then $f^{-1}(\overline{\gamma}_V)$ is the In-val-fuz- z-subalgebra of G.

Proof. Consider the In-val-fuz-$\overline{\varphi}$- translation $\overline{\gamma}_v$ of V being the In-val-fuz- z-subalgebra of V.

Let u, v $\in$ V; then:

$f^{-1}(\overline{\gamma}^{\overline{\mathrm{T}}}_{v\overline{\varphi}})(u * v) = f^{-1}(\overline{\gamma}_V)(u * v) + \overline{\varphi}$

$= \overline{\gamma}_V\,(z(u * v) + \overline{\varphi})$

$= \overline{\gamma}_V\,(z(u) * z(v) + \overline{\varphi}$

$\geq$ rmin$\{\overline{\gamma}_V\,(z(u) + \overline{\varphi}), \overline{\gamma}_V\,(z(v) + \overline{\varphi})\}$

= rmin$\{f^{-1}(\overline{\gamma}^{\overline{\mathrm{T}}}_{v\overline{\varphi}})(u), f^{-1}(\overline{\gamma}^{\overline{\mathrm{T}}}_{v\overline{\varphi}})(v)\}$

Hence, $f^{-1}(\overline{\gamma}_V)$ is an In-val-fuz-z-subalgebra of G.

Theorem 3.12. Assume that U and V are two z-algebra, and f: U → V is a epi-morphism. If the In-val-fuz-$\overline{\varphi}$-translation $\overline{\gamma}_U$ of U is an In-val-fuz-z-subalgebra of U, then $f^{-1}(\overline{\gamma}_U)$ is an In-val-fuz-z-subalgebra of G.

Proof. Let the In-val-fuz-$\overline{\varphi}$-translation $\overline{\gamma}_U$ of U is the In-val-fuz- z-subalgebra of U.

Take u, v ∈ U; then:

$f^{-1}(\overline{\gamma}_{U_{\overline{\varphi}}}^{\overline{T}})\,(u * v) = f^{-1}(\overline{\gamma}_U)(u * v) + \overline{\varphi}$

$= \overline{\gamma}_U\ (z(u * v)\ +\ \overline{\varphi})$

$= \overline{\gamma}_U\,(z(u) * z(v) + \overline{\varphi}$

$\geq \mathrm{rmin}\{\overline{\gamma}_U\,(z(u) + \overline{\varphi}, \overline{\gamma}_v\,(z(v) + \overline{\varphi}\}$

$= \mathrm{rmin}\{f^{-1}(\overline{\gamma}_{U_{\overline{\varphi}}}^{\overline{T}})\,(u),\ f^{-1}(\overline{\gamma}_{U_{\overline{\varphi}}}^{\overline{T}})\,(v)\}$

Therefore, $f^{-1}(\overline{r}_U)$ is an In-val-fuz-z-subalgebra of G.

Definition 3.13. Consider A and B as any two fuzzy sets that contain membership functions μ_A and μ_B, respectively. Following that, the Cartesian product A× B with membership function $\mu_{A\times B}$: G × G → [0, 1] is characterized by $\mu_{A\times B}(x,\ y) = \min\{\mu_A(x), \mu_B(y)\}$ ∀x, y ∈ G.

Theorem 3.14. Consider $\overline{\gamma}_1$ and $\overline{\gamma}_2$ are In-val-fuz-z-subalgebras of G. Let $\overline{T} = \mathrm{rmin}\{\overline{T}_{\overline{\gamma}_1}, \overline{T}_{\overline{\gamma}_2}\}$, where $\overline{T}_{\overline{\gamma}_1} = \{[1, 1] - \mathrm{rsup}\ \{\overline{\gamma}_1(x); z \in Z\}\}'$ and $\overline{T}_{\overline{\gamma}_2} = \{[1, 1] - \mathrm{rsup}\{\overline{\gamma}_2(x); z \in Z\}\}'$. Let $\overline{\varphi} \in [\overline{0}, \overline{T}]$. Then, the $\overline{\varphi}$- translation of the Cartesian product $\overline{\gamma}_1 \times \overline{\gamma}_2$ of $\overline{\gamma}_1$ and $\overline{\gamma}_2$ is an In-val-fuz-z-subalgebra of G × G.

Proof. Let $\overline{\gamma}_1$ and $\overline{\gamma}_2$ be two In-val-fuz-z-subalgebras of a z-algebra G, and:

$\overline{\varphi} \in [\overline{0}, \overline{T}]$.

Now, by Theorem 3.4, $\overline{\gamma}_{1\overline{\varphi}}^{\overline{T}}$ and $\overline{\gamma}_{2\overline{\varphi}}^{\overline{T}}$ are In-val-fuz-z-subalgebras of G.

Clearly, $\overline{\gamma}_{1\overline{\varphi}}^{\overline{T}} \times \overline{\gamma}_{2\overline{\varphi}}^{\overline{T}}$ is an In-val-fuz-z-subalgebra of G× G. Further:

$(\overline{\gamma}_1 \times \overline{\gamma}_2)_{\overline{\varphi}}^{\overline{T}}(a, b) = (\overline{\gamma}_1 \times \overline{\gamma}_2)\,(a, b) + \overline{\varphi}$

$\geq \mathrm{rmin}\{\overline{\gamma}_1(a), \overline{\gamma}_1(b)\} + \overline{\varphi}$

$= \mathrm{rmin}\{\overline{\gamma}_1\,(a) + \overline{\varphi}\,, \overline{\gamma}_2(b) + \overline{\varphi}\}$

$= \mathrm{rmin}\{\overline{\gamma}_{1\overline{\varphi}}^{\overline{T}}\,(a), \overline{\gamma}_{2\overline{\varphi}}^{\overline{T}}(b)\}$

$= (\overline{\gamma}_{1\overline{\varphi}}^{\overline{T}} \times \overline{\gamma}_{2\overline{\varphi}}^{\overline{T}})\,(a, b)$

Hence, $(\overline{\gamma}_1 \times \overline{\gamma}_2)_{\varphi}^{\overline{T}}$ is an In-val-fuz- z-subalgebra of G.

INTERVAL-VALUED-FUZZY $\overline{\mu}$-MULTIPLICATION OF Z-SUBALGEBRA

The notation of In-val-fuz-$\overline{\mu}$-multiplication is introduced in this section. Also, we explain some examples and provide some straightforward results to demonstrate the concept.

Definition 4.1. If $\overline{\gamma}$ is an In-val-fuz subset of G and $\overline{\mu} \in$ C[0, 1], and the map $\overline{\mu}$: Z → C[0, 1] is said to be an In-val-fuz-$\overline{\mu}$-multiplication of $\overline{\gamma}$, then it is satisfying

$\overline{\gamma}_{\overline{\mu}}^{N}(z) = \overline{\mu}\,.\overline{\gamma}(z)$, ∀ z∈G.

Example 4.2. By the previous Example 3.2, take $\overline{\mu}$ = [0.1, 0.3]. Following that, $\overline{\mu}$-multiplication of In-val-fuz set $\overline{\gamma}$ is as follows:

$\overline{\gamma}_{\overline{\mu}}^{N}(0) = [0.03, 0.15]$, $\overline{\gamma}_{\overline{\mu}}^{N}(\alpha) = \overline{\gamma}_{\overline{\mu}}^{N}(\delta) = [0.02, 0.18]$ and $\overline{\gamma}_{\overline{\mu}}^{N}(\beta) = [0.01, 0.21]$.

Theorem 4.3. For every In-val-fuz- z-subalgebra $\overline{\gamma}$ of G and $\overline{\mu} \in$ C[0, 1], then the In-val-fuz $\overline{\mu}$-multiplication $\overline{\gamma}_{\overline{\mu}}^{N}$ (z) of $\overline{\gamma}$ is also an In-val-fuz-z-subalgebra of G.

Proof. Assume u, v ∈ Z, and $\overline{\mu} \in$ C[0, 1].

Then, $\overline{\gamma}(u * v) \geq \mathrm{rmin}\ \{\overline{\gamma}(u), \overline{\gamma}(v)\}$.

Now, $\overline{\gamma}_{\overline{\mu}}^{N}(u * v) = \overline{\mu}\,.\,\overline{\gamma}(u * v)$

$\geq\ \overline{\mu}\,.\,\mathrm{rmin}\{\overline{\gamma}(u), \overline{\gamma}(v)\}$

$= \mathrm{rmin}\ \{\overline{\mu}\,.\,\overline{\gamma}(u), \overline{\mu}\,.\,\overline{\gamma}(v)\}$

$= \mathrm{rmin}\{\overline{\gamma}_{\overline{\mu}}^{N}(u), \overline{\gamma}_{\overline{\mu}}^{N}(v)\}$

Hence, $\overline{\gamma}_{\overline{\mu}}^{N}$ of $\overline{\gamma}$ is an In-val-fuz-z-sub algebra of G.

Now, using the converse of the previous theorems, it is as follows.

Theorem 4.4. For every In-val-fuz-z-subalgebra $\bar{\gamma}$ of G and $\bar{\mu} \in C[0, 1]$, if the In-val-fuz $\bar{\mu}$-multiplication $\bar{\gamma}^{N}_{\bar{\mu}}(z)$ of $\bar{\gamma}$ is an In-val-fuz-z-subalgebra of G, then is also an $\bar{\gamma}$.

Proof. Take u,v ∈ G:

Consider that $\bar{\gamma}^{N}_{\bar{\mu}}(z)$ of $\bar{\gamma}$ represents an In-val-fuz-z-sub algebra of G for some $\bar{\mu} \in C[0, 1]$.

$$\bar{\mu} \,.\, \bar{\gamma}(u * v) = \bar{\gamma}^{N}_{\bar{\mu}}(u * v)$$

$$\geq \mathrm{rmin}\{\bar{\gamma}^{N}_{\bar{\mu}}(u), \bar{\gamma}^{N}_{\bar{\mu}}(v)\}$$

$$= \mathrm{rmin}\{\bar{\mu} \,.\, \bar{\gamma}(u), \bar{\mu} \,.\, \bar{\gamma}(v)\}$$

$$= \bar{\mu} \,.\, \mathrm{rmin}\ \{ \bar{\gamma}(u), \bar{\gamma}(v) \}$$

Hence, $\bar{\gamma}^{N}_{\bar{\mu}}(z)$ of $\bar{\gamma}$ is the In-val-fuz-z-subalgebra of G.

Theorem 4.5. Assuming U and V are two z-algebra, and f: U → V is a homomorphism. If the In-val-fuz $\bar{\mu}$-multiplication $\bar{\gamma}^{N}_{\bar{\mu}}(z)$ of V is a In-val-fuz z-subalgebra of V, then $f^{-1}(\bar{\gamma}_V)$ is the In-val-fuz- z-subalgebra of G.

Proof. Consider that the In-val-fuz-$\bar{\mu}$-multiplication $\bar{\gamma}^{N}_{\bar{\mu}}$ (z) of V is the In-val-fuz- z-subalgebra of V.

Let u, v ∈ V; then:

$$f^{-1}(\bar{\gamma}^{N}_{\bar{\mu}})(u * v) = f^{-1}(\bar{\gamma}_V)(u * v) + \bar{\varphi}$$

$$= \bar{\gamma}_V\ (z(u * v) + \bar{\varphi})$$

$$= \bar{\gamma}_V\ (z(u) * z(v) + \bar{\varphi}$$

$$\geq \mathrm{rmin}\{\bar{\gamma}_V\ (z(u) + \bar{\varphi}), \bar{\gamma}_V\ (z(v) + \bar{\varphi})\}$$

$$= \mathrm{rmin}\{f^{-1}(\bar{\gamma}^{N}_{\bar{\mu}})\ (u), f^{-1}(\bar{\gamma}^{N}_{\bar{\mu}})\ (v)\}$$

Hence, $f^{-1}(\bar{\gamma}_V)$ is an In-val-fuz-z-subalgebra of G.

Theorem 4.6. Assume U and V are two z-algebra, and f: U → V is a homomorphism. If the In-val-fuz $\bar{\mu}$– multiplication $\bar{\gamma}^{N}_{\bar{\mu}}(z)$ of V is a In-val-fuz z-subalgebra of V, then $f^{-1}(\bar{\gamma}_V)$ is the In-val-fuz- z-subalgebra of G.

Proof. Consider that the In-val-fuz-$\bar{\mu}$– multiplication $\bar{\gamma}^{N}_{\bar{\mu}}$ (z) of V is the In-val-fuz- z-subalgebra of V.

Let u, v ∈ V; then:

$$f^{-1}(\bar{\gamma}^{\bar{T}}_{V_{\bar{\varphi}}})(u * v) = f^{-1}(\bar{\gamma}_V)(u * v) + \bar{\varphi}$$

$$= \bar{\gamma}_V\ (z(u * v) + \bar{\varphi})$$

$$= \bar{\gamma}_V\ (z(u) * z(v) + \bar{\varphi}$$

$$\leq \mathrm{rmax}\ \{\bar{\gamma}_V\ (z(u) + \bar{\varphi}), \bar{\gamma}_V\ (z(v) + \bar{\varphi})\}$$

$$= \mathrm{rmax}\{f^{-1}(\bar{\gamma}^{\bar{T}}_{V_{\bar{\varphi}}})\ (u), f^{-1}(\bar{\gamma}^{\bar{T}}_{V_{\bar{\varphi}}})\ (v)\}$$

Hence, $f^{-1}(\bar{\gamma}_V)$ is an In-val-fuz-z-subalgebra of G.

Theorem 4.7. For every In-val-fuz- z-subalgebra $\bar{\gamma}$ of G and $\bar{\mu} \in C[0, 1]$, the In-val-fuz $\bar{\mu}$- multiplication $\bar{\gamma}^{N}_{\bar{\mu}}(z)$ of $\bar{\gamma}$ is an In-val-fuz-z-subalgebra of G.

Proof. Take u, v ∈ G, and $\bar{\mu} \in C[0, 1]$.

Then, $\bar{\gamma}(u * v) \geq \mathrm{rmin}\ \{\bar{\gamma}(u), \bar{\gamma}(v)\}$

Now, $\bar{\gamma}^{N}_{\bar{\mu}}(u * v) = \bar{\gamma}(u * v) + \bar{\varphi}$

$$\geq\ \mathrm{rmin}\{\bar{\gamma}(u), \bar{\gamma}(v)\}\bar{\varphi}$$

$$= \mathrm{rmin}\{\bar{\gamma}(u) + \bar{\varphi}\ , \bar{\gamma}(v) + \bar{\varphi}\}$$

$$= \mathrm{rmin}\{\bar{\gamma}^{N}_{\bar{\mu}}(u), \bar{\gamma}^{N}_{\bar{\mu}}(v)\}$$

Thus, $\bar{\gamma}^{N}_{\bar{\mu}}$ of $\bar{\gamma}$ is an In-val-fuz-z-subalgebra of G.

Theorem 4.8. Let $\bar{\gamma}$ be an In-val-fuz subset of G, $\bar{\mu} \in C[0, 1]$ and $\bar{\varphi} \in [\bar{0}, \bar{T}]$. The mapping $\bar{\gamma}^{N\bar{T}}_{\bar{\mu}\bar{\varphi}}$: G → C[0, 1] represents an In-val-fuz-magnified $\bar{\mu}\bar{\varphi}$-translation of $\bar{\gamma}$. Then, $\bar{\gamma}$ is an In-val-fuz-z-subalgebra of G if it is an In-val-fuz-z-subalgebra of G.

Proof. Consider $\bar{\gamma}$ an In-val-fuz subset of Z, $\tilde{\mu} \in C[0, 1]$ and $\bar{\varphi} \in [\bar{0}, \bar{T}]$. The mapping $\bar{\gamma}^{N\bar{T}}_{\tilde{\mu}\bar{\varphi}}$: Z → C[0, 1] is an In-val-fuz-magnified $\tilde{\mu}\bar{\varphi}$-translation of $\bar{\gamma}$.

Let $\overline{\gamma}$ be an In-val-fuz subset of G.

Following this:

$$\overline{\gamma}(u * v) \geq \mathrm{rmin}\{\overline{\gamma}(u), \overline{\gamma}(v)\}$$

Now:

$$\overline{\gamma}^{N\overline{T}}_{\overline{\mu}\overline{\varphi}}(u * v) = \overline{\mu} \,.\, \overline{\gamma}\,(u * v) + \overline{\varphi}$$

$$\geq \overline{\mu} \,.\, \mathrm{rmin}\{\overline{\gamma}(u), \overline{\gamma}(v)\}\overline{\varphi}$$

$$= \mathrm{rmin}\,\{\overline{\mu} \,.\, \overline{\gamma}(u) + \overline{\varphi}, \overline{\mu} \,.\, \overline{\gamma}(v) + \overline{\varphi}\}$$

$$= \mathrm{rmin}\{\overline{\gamma}^{N\overline{T}}_{\overline{\mu}\overline{\varphi}}\,(u), \overline{\gamma}^{N\overline{T}}_{\overline{\mu}\overline{\varphi}}(v)\}$$

Hence, $\overline{\gamma}^{N\overline{T}}_{\overline{\mu}\overline{\varphi}}$ of $\overline{\gamma}$ is an In-val-fuz-z-subalgebra of G.

Consider $\overline{\gamma}^{N\overline{T}}_{\overline{\mu}\overline{\varphi}}$ (u) of $\overline{\gamma}$ an In-val-fuz-z-subalgebra of G.

And then:

$$\overline{\mu} \,.\, \overline{\gamma}(u * v) + \overline{\varphi} = \overline{\gamma}^{N\overline{T}}_{\overline{\mu}\overline{\varphi}}(u * v)$$

$$\geq \mathrm{rmin}\{\overline{\gamma}^{N\overline{T}}_{\overline{\mu}\overline{\varphi}}\,(u), \overline{\gamma}^{N\overline{T}}_{\overline{\mu}\overline{\varphi}}(v)\}$$

$$= \mathrm{rmin}\{\overline{\mu} \,.\, \overline{\gamma}(u) + \overline{\varphi}, \overline{\mu} \,.\, \overline{\gamma}(v) + \overline{\varphi}\}$$

$$= \overline{\mu} \,.\, \mathrm{rmin}\{\overline{\gamma}(u), \overline{\gamma}(v)\} + \overline{\varphi}$$

Hence $\overline{\gamma}$ is an In-val-fuz-z-subalgebra of G.

DISCUSSIONS AND IMPORTANCE OF THIS STUDY

Expanding on the insightful conclusions drawn from this study, our deeper understanding of interval-valued fuzzy sets and their translation and multiplication within z-subalgebras of z-algebras sets the stage for continued exploration and refinement of these concepts. The potential applications of these fuzzy operations in fields such as artificial intelligence, decision-making, and engineering underscore their practical relevance in managing uncertainty and complexity. As we envision the future trajectory of this research, it becomes evident that our study establishes a solid foundation for further investigations and application development in the domain of interval-valued fuzzy algebra within z-subalgebras of z-algebras. The discovered generalization of BCK/BCI/G/PS-algebras to z-algebras opens up new avenues for theoretical advancements, whereas the exploration of fuzzy extensions in z-subalgebras adds a dimension that can be applied across various algebraic structures. This innovative approach promises to yield novel findings and contribute significantly to our evolving understanding of algebraic systems. The study's findings set the stage for forthcoming research endeavors, where the application of these concepts in diverse mathematical frameworks holds the potential for groundbreaking discoveries and practical implementations.

CONCLUSION

Through this study, we have covered a deeper understanding of how interval-valued fuzzy sets can be translated and multiplied within the context of z-subalgebras of z-algebras. As we move forward, it is essential to continue exploring and refining these concepts, potentially leading to practical applications in fields such as artificial intelligence, decision-making, and engineering, where fuzzy logic and algebraic structures play a vital role in managing uncertainty and complexity. This study serves as a foundation for further research and application development in the domain of interval-valued fuzzy algebra within z-subalgebras of z-algebras. A further generalization of BCK/BCI/G/PS-algebras has been found to the z-algebra. Interestingly, a fuzzy extension of z-subalgebra of z-algebra has been studied, which adds another dimension to the defined z-algebras. This idea can also be applied in various algebraic structures to produce novel findings in our upcoming research.

FUTURE WORK

In the realm of mathematical structures, this research lays the foundation for the practical application of interval-valued fuzzy translation and multiplication across various domains. The inherent versatility of these fuzzy operations prompts an exploration into extending established structures, encompassing crucial mathematical concepts such as homomorphism, endomorphism, p-ideals, and beyond. Future research endeavors could delve into understanding how interval-valued fuzzy translation and multiplication play a transformative role in preserving algebraic structures between different mathematical systems – investigating their behavior within the same structure in the context of endomorphisms, and exploring their interaction with the properties of p-ideals. This extension not only broadens the scope of fuzzy operations but also opens avenues for practical applications and theoretical advancements, hence shedding light on unexplored territories and providing a rich ground for further exploration in the mathematical landscape.

REFERENCES

ALSHERI HA. 2021. Fuzzy translation and fuzzy multiplication in BRK-algebras. European Journal of Pure and Applied Maths 14(3): 737–745.

AUB AYNB ANSARI M, CHANDRAMOULEESWARAN M. 2014. Fuzzy translation of fuzzy β-subalgebras of β-algebras. International Journal of Pure and Applied Maths 92(5): 657–667.

BARBIYA SR. 2015. Fuzzy translation and fuzzy multiplication of IVF BG-algebras. International Journal of Mathematical Archive 6: 25–32.

CHANDRAMOULEESWARAN M. 2017. A Note on Z-algebras. Italian Journal of Pure and Applied Maths 38: 707–714.

HEMAVATHI P, MURALIKRISHNA P, PALANIVEL K. 2016. Study on interval valued fuzzy translation and multiplication of interval valued fuzzy β-subalgebra. International Journal of Pure and Applied Maths 109(2): 245–256.

IAMPAN A. 2017. Fuzzy translation of a fuzzy set in UP-algebras. J Indones Math Soc 23(2): 1–19.

JANA C, PAL M, SAIED AB. 2017. $(\in, \in \vee q)$-bipolar Fuzzy BCK/BCI-algebras. Missouri Journal of Mathematical Sciences 29(2): 139–160.

JANA C, SENAPATI T, PAL M. 2016. (∈, ∈∨ q)-intuitionistic fuzzy BCI-subalgebras of a BCI-algebra: Journal of Intelligent and Fuzzy Systems 31(1): 613–621.

JANA C, PAL M. 2017a. Generalized intuitionistic fuzzy ideals of BCK/ BCI-algebras based on 3-valued logic and its computational study. Fuzzy Information and Engineering 9(4): 455–478.

JANA C, PAL M. 2017b. Application of (α, β)-soft intersectional sets on BCK/BCI-algebras. International Journal of Intelligent Systems Technologies and Applications 16(3): 269–288.

JANA C, SENAPATI T, SHUM KP, PAL M. 2019. Bipolar fuzzy soft subalgebras and ideals of BCK/BCI-algebras based on bipolar fuzzy points. Journal of Intelligent and Fuzzy Systems 37(2): 2785–2795.

JUN Y-B, KIM H-S. 2009. Bipolar fuzzy translation in BCK/BCI-algebras. Journal of the Chungcheong Mathematical Society 22(3): 399–408.

KHALID M, JUN YB. 2020. Magnification of MBJ-neutrosophic translation on G-algebra. International Journal of Neutrosophic Science 2(1): 27–37.

KHALID M. 2019. Intuitionistic fuzzy translation and multiplication of G-algebra. Journal of Fuzzy Mathematics 27(3): 543–559.

PRASANA M, PREAMKUMAR M, ISMAIL S. 2021. Fuzzy translation and multiplication in BG-algebras. AIP Conference Proceedings 2261(1): 030090.

PRIYA T, RAMACHANDRAN T. 2014. Fuzzy translation and fuzzy multiplication on PS-algebras. International Journal of Innovation in Science and Mathematics 2(5): 485–489.

PRASANA A, PREAMKUMAR M. 2020. Algebraic properties on w-fuzzy translation and multiplication in BH-algebras. AIP Conference Proceedings 5: 0300901–0300907.

SOWMIYA S, JEYALAKSHMI P. 2020a. Z-homomorphism and Cartesian product on fuzzy α-translation and fuzzy β-multiplication of z-algebras. AIP Conference Proceedings 2261(1): 030098.

SOWMIYA S, JEYALAKSHMI P. 2020b. Fuzzy α-translation and fuzzy β-multiplication of z-algebras. Advances in Mathematics Scientific Journal 9(3): 1287–1292.

SUJATHA K, MURALIKRISHNA P. 2015. Intuitionistic fuzzy α-Translation on β-algebras. International Journal of Pure and Applied Mathematics 98(5): 39–44.

ZADEH LA. 1965. Fuzzy sets. Inform Control 8(3): 338–353.

ZADEH LA. *1975*. The concept of a linguistic variable and its application to approximate reasoning: Information Sci 8: 199–249