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Pricing a Combined Life and Health Insurance with Level Premiums and Varying Benefits

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The growing interest in insurance poses a threat to insurance companies. As the number of individuals seeking insurance coverage increases, the number of individuals adversely selecting the insurer also increases, which can lead to the insurance product being unprofitable. Thus, this study explores designing an insurance product – a combined life insurance and health insurance product that will reflect the actual claims of an individual to his/her policy – that will hopefully help prevent unprofitability of the insurance product. This is done by applying incentives and penalization, or the bonus-malus system, to the insurance benefit while leaving the premium constant. Assuming that premiums for life insurance and health insurance are constant, transition and pricing models are derived. Considering two forms of combination of life and health insurance: a combined life and hospitalization income insurance and a combined life and medical insurance, we illustrate the derived model assuming Makeham distribution for life insurance claims and Poisson distribution and gamma distribution for the hospitalization income insurance and medical insurance claims, respectively. The proposed design gives a new framework for combining life and health insurance, as well as a methodology for combining other types of insurances.

Keywords: bonus-malus system, gamma process, Poisson process

INTRODUCTION

The insurance industry is growing in the Philippines – from life insurance to health insurance, to automobile insurance, to agricultural insurance, and so on. However, this growing interest in insurance poses a problem to the insurer. As the number of individuals seeking insurance coverage increases, the number of individuals adversely selecting the insurer also increases. Adverse selection happens when the true risk posed by insureds in a heterogenous group is not fully known to the insurer and may lead to the insurer facing larger than expected payouts (Di Novi 2011). This may then lead to the unprofitability of the insurance policy. It is not easy to study and determine the true risk posed by each insured, and studying this may also incur the insurer some costs. Insurers usually set an initial price and adjust this price according to the experience (Kafkova 2015). However, using this method, some low-risk people decide to drop out of the insurance pool as the premium may be too high than intended for their risk class. This will result in the insurance pool being filled with the high-risk people because of their higher willingness to pay (Geruso and Layton

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2017), which in the end will bring further losses to the insurer (Cummins et al. 1983; Di Novi 2011; Geruso and Layton 2017).

To rectify this situation, there must be a system that can reflect the actual claims of the policyholder to his/her own insurance to the premium. This is the goal of the bonus-malus system. The bonus-malus system was introduced by Lemaire in 1995 in automobile insurance (Lemaire 1995). This system either reduces or increases the premium depending on the previous year's experience. Bonus stands for the incentive given to the policyholder whenever there is no claim or there is an unsubstantial amount of claim within the given period. This results in a decrease in premium in the next renewal of the policy. On the other hand, malus stands for penalization whenever there is a substantial claim, e.g. claims if the car underwent damage repairs, resulting in an increase in premium the following year (Teodorescu 2012).

Kafkova (2015) mentioned that to implement the bonus-malus system, a policyholder is first given an a priori premium, which is usually based on the make of the car, the engine, and other factors that could possibly affect the claims. Then, after the policy period, the insurer reclassifies the policyholder and gives an a posteriori premium that is now based on the policyholder's claim experience. Several automobile insurance policies utilizing the bonus-malus systems were compared. A new bonus-malus system was made, and it was found out that this design is more stringent than existing ones. In the new design, each claim is penalized by going one class down. In the designs used by insurance companies, each claim is penalized by going two or more classes down. However, the premium paid by more risky drivers is less compared to when using the new design made. It is concluded that this is because insurers aim for the marketability of their product.

Ibiwoye et al. (2011) mentioned that the bonus-malus system is determined by three elements – namely, the premium scale, initial class, and transition rules. These elements are used to determine how the insured transitions from one class to another, to compute for the right premium, and to apply rewards or penalization.

Pitrebois et al. (2003) provided an analytic derivation of the model by Taylor (1997). Analytic formulae for transition probabilities and premium were made, and illustrations were provided.

Cummins et al. (1983) mentioned that high-risk individuals continue to purchase the insurance product despite the increase in premium as they need the coverage. However, the individuals with less or average risks may find the increase in the premium too heavy and may decide to drop out of the policy. Xiao and Meng (2007) proposed a model for a financially balanced bonus-malus system incorporating the tolerance level of the insured.

Some researchers have also suggested ways to reduce adverse selection. Geruso and Layton (2017) investigated the most common ways of premium rating – namely, consumer subsidies or penalties to take-up insurance, risk adjustment, and contract regulation. Frank et al. (1997) mentioned risk adjustment, carving out of benefits, and cost-sharing. Carving out benefits and cost-sharing were found to be more efficient in reducing adverse selection. Cummins et al. (1983) and Di Novi (2011) studied offering separate insurance policies for different risk classes.

There is little to no literature that states how the bonus-malus system is implemented in health insurance. However, the application is apparent to the increase in premium whenever there is a sizable claim in the previous year. To reduce the chance of having unprofitable insurance, this study proposes a way to penalize the individuals adversely selecting the insurer other than increasing the premium. The focus of this study is to design a combined life and health insurance policy by applying the incentives and penalization to the life and health insurance benefits leaving the premium constant. In other words, this insurance will have both life and health benefits, and the insured will pay the same premium each year though the benefits will vary depending on the insured's claim experience. This study will focus on designing insurance with a fixed premium but varying benefits.

The remaining sections of the paper are organized as follows. Section 2 presents the theoretical framework of the study where the bonus-malus system framework – through the transition probability matrix – will be presented. A brief summary of the premium calculation principle will also be given. In Section 3, the Results and Discussion, the premium formula of the combined life and health insurance will be derived. There are two forms of combining life and health insurance considered in this paper – a combined life and hospitalization income insurance, and a combined life and medical insurance. Finally, in Section 4, numerical illustrations of the model with defined transition rules and states and derived transition probabilities for each model will be presented.

THEORETICAL FRAMEWORK

In this section, models pertinent to our study – such as the transition probability matrix, the actuarial notations, and the life-contingent models used for premium calculation – are given.

Transition Probability Matrix

A bonus-malus system can be modeled as a discrete Markov chain, which can be described by a transition probability matrix (Ismail 2020). Suppose there are n states in a bonus-malus system, where each state has its own corresponding life and health insurance benefits, and let T be the transition probability matrix. The entries of the transition probability matrix give the probability of the insured transferring from one state to another state depending on the claims made from the health insurance. Define $T = [t_{ij}]$, where t_{ij} is the probability that an insured will transition from state i to state j in one period, $i, j = 0, \ldots, n-1$. In matrix notations, we have:

$$T = \begin{pmatrix} t_{00} & t_{01} & \cdots & t_{0\,n-1} \\ t_{10} & t_{11} & \cdots & t_{1\,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n-1\,0} & t_{n-1\,1} & \cdots & t_{n-1\,n-1} \end{pmatrix}. \tag{1}$$

The entries of the transition probability matrix are dependent on the probability distribution of claims and on the transition rules defining the bonus-malus system.

Also, note that an insured from state i will either remain in state i or transition to another state. Hence, the sum of the entries in each row should be equal to 1.

In general, define $t_{ij}^{(m)}$ as the probability that an insured will transfer from state i to state j after m periods, then:

$$t_{ij}^{(m)} = \sum_{k_1=0}^{n-1} \sum_{k_2=0}^{n-1} \cdots \sum_{k_{m-1}=0}^{n-1} t_{ik_{m-1}} t_{k_{m-1}k_{m-2}} \cdots t_{k_2k_1} t_{k_1j} \tag{2}$$

(Denuit *et al.* 2007). The probabilities of all possible paths from state i to state j after m periods are reflected in the formula. In addition, $t_{0i}^{(0)} = \mathbb{I}(i = 0) = \begin{cases} 1 & \text{if } i = 0 \\ 0 & \text{otherwise} \end{cases}$.

Premium Calculation

The following are the survival models and general models for life insurance, life annuities, and premium. These models are from Dickson *et al.* (2009) and Bowers *et al.* (1997).

Let (x) denote a life aged x, $x \geq 0$. The death of (x) can occur at any age greater than x. We denote by a continuous random variable T_x, the future lifetime of (x). The actuarial notations for survival and mortality probabilities are:

$${}_tp_x = \mathbb{P}(T_x > t) \tag{3}$$

which denotes the probability that (x) survives to at least age $x + t$, and:

$${}_tq_x = \mathbb{P}(T_x \leq t) \tag{4}$$

which denotes the probability that (x) dies before age $x + t$.

The curtate future lifetime random variable is defined as the integer part of the future lifetime and is denoted by K_x for a life aged x. We can think of it as the number of whole years lived by an individual. We can find the probability function of K_x by noting that for $k = 0,1,2, \ldots, K_x = k$ if and only if (x) dies between ages $x + k$ and $x + k + 1$. Thus, for $k = 0,1,2, \ldots$:

$$\mathbb{P}(K_x = k) = \mathbb{P}(k \leq T_x < k+1) = \ {}_kp_x q_{x+k}. \tag{5}$$

Now, consider insurance with unit benefit payable at the end of the year of death. Let v^{k+1} be the discount factor required for the period from the time of payment back to the time when the policy is issued, where k is the insured's curtate future lifetime. Then, the actuarial present value for this insurance is:

$$A_x = \sum_{k=0}^{+\infty} v^{k+1}\ {}_kp_x q_{x+k}. \tag{6}$$

The summation runs for all possible values of the curtate future lifetime.

Next, consider an annuity that pays a unit amount at the beginning of each year that the annuitant (x) survives. Then, the actuarial present value of the annuity is:

$$\ddot{a}_x = \sum_{k=0}^{+\infty} v^k\ {}_kp_x. \tag{7}$$

The summation runs for all possible times of payment in the annuity.

Last, denote by P_x the premium of insurance payable at the end of the policy year of death, where premiums are payable at the beginning of each policy year for as long as the insured lives. Then:

$$P_x = \frac{A_x}{\ddot{a}_x}. \tag{8}$$

Makeham Survival Model

The Makeham survival model will be used in this study. The probability that a newborn child will survive x years using the Makeham survival model is given by:

$$S_0(x) = \exp\left(-Ax - \frac{B}{\log c}(c^x - 1)\right), \tag{9}$$

where $A > 0$, $0 < B < 1$ and $c > 1$ (Dickson *et al.* 2009). This model shows that the rate of mortality increases exponentially with age, which can be described or fitted by the parameters that will be used. Parameter A represents the age-independent mortality, and parameters B and c cover the increase in the probability of mortality as one grows older (Kirkwood 2015).

The probability that a life aged x survives up to age $x+m$ is:

$${}_mp_x = \ \exp\left(-Am - \frac{B}{\log c}c^x(c^m - 1)\right) \tag{10}$$

and the probability that a life aged x survives up to age $x+m$ but dies before age $x+m+1$ is:

$${}_mp_x q_{x+m} = \widehat{K}\exp\left(-Am - \frac{B}{\log c}c^x(c^m - 1)\right), \tag{11}$$

where $\widehat{K} = q_{x+m} = 1 - \exp\left(-A - \frac{B}{\log c}m(c-1)\right)$.

The standard ultimate survival model, a survival model commonly used in actuarial studies, is a Makeham survival model with $A = 0.00022$, $B = 2.7 \times 10^{-6}$, and $c = 1.124$. This model with the mentioned parameters will be used in the illustrations.

RESULTS AND DISCUSSION

Consider an insurance policy with a death benefit payable at the end of the year of death. In this very same insurance, another feature is added: this insurance also pays for the medical expenses of the life insured, subject to a maximum benefit level, for as long as the policy is in force, *i.e.* as long as the insured still lives and pays the required premium.

This insurance is designed by using the principle of the bonus-malus system; however, instead of applying the rewards and penalization on the premium, we apply the principle on the benefits. That is, there are provisions for the maximum benefit of the insurance to increase after a year where the insured makes little claims, if at all, from the health insurance (good years); there are provisions as well for the decrease in the insurance benefits whenever the insured makes too many claims on the health insurance during the previous year (bad years). The criteria for the change in benefits can be specified by the insurer and will be reflected in the transition rules.

To design this insurance, we need to set-up the transition rule for going up or down states based on the claims from the health insurance, and to derive the corresponding actuarial present value for life and health insurance.

The Transition Rule

Upon purchase of the insurance, all insured starts at state 0. In this state, the highest possible life insurance benefit and health insurance benefit can be enjoyed by the insured. After one policy year, the insurer evaluates the insured's health insurance performance and changes the insured's state based on it. Let $\mathcal{H}_m$ be the claims made by the insured for year m. Note that $\mathcal{H}_m$ can be in terms of the number of times a health insurance claim was made or the total amount claimed from the health insurance, depending on the type of the health insurance. Suppose that there are n possible states $(0,1,\ldots,n-1)$, and denote by $S_{i\to j}$ the rule for transitioning from state i to state j. The transition rule $S_{i\to j}$ may vary depending on the insurer and on the distribution of the claims. Using the transition rule and distribution of $\mathcal{H}_m$, transition probabilities can be computed.

The probability of transitioning from state i to state j is denoted by t_{ij}. The notation t_{02} indicates the probability that a life insured which started from state 0 will transition to state 2 after one year. This implies that the life insured made substantial claims to the health insurance and is now penalized by going down states. In effect, the insured will receive less benefit in the next year. Accordingly, the notation $t_{02}^{(3)}$ indicates the probability that the insured, which started from state 0 will end in state 2 after three years. This again indicates that the life insured made substantial claims in the health insurance. In contrast to this, the notation $t_{21}^{(3)}$ indicates that the life insured transitioned from state 2 to state 1 after three years, implying that the life insured had good years and is rewarded by an increase in benefits.

Model for the Combined Life and Health Insurance

The derivations for the actuarial present value of the life benefit and the health benefit are different. The life benefit is paid by the insurer to the insured at the end of the year of death, but the health benefit is paid to the insured whenever the insured seeks medical services while alive. We first start with the derivation of the actuarial present value of the life benefit and then give the derivation of the actuarial present value of the two types of health insurance considered in this study.

Actuarial Present Value of the Death Benefit

Consider life insurance with benefit payable at the end of the policy year. Let $B_i, i = 0,1,\ldots,n-1$, where B_0 is the original face amount of the life insurance, be the corresponding death benefit if the insured dies at state i.

Suppose that the insured is aged x upon purchase of the insurance. Let random variable Z_i be the present value of the death benefit if the insured dies at state i.

Note that if the insured survives to age $x+m$ but dies upon attaining that age, then the insured will receive the benefit depending on the state he/she belongs to during that policy year. Say the insured dies at state i, and receives the benefit of B_i, we have:

$$Z_i = B_i v^{m+1} \tag{12}$$

for $m = 0,1,2,\dots$.

We are now concerned not only at the time of death but also about the state in which the insured is at the time of death. We assume that the future lifetime and distribution of claims, $\mathcal{H}_m$, are independent. Note that $t_{0i}^{(m)}$ is the probability that the insured is at state i, m years after purchasing the insurance. Hence, the probability that the insured life (x) dies between ages $x+m$ and $x+m+1$ in state i is ${}_m p_x q_{x+m} t_{0i}^{(m)}$.

Now, similar to Equation 6, the actuarial present value of the death benefit if the insured dies at state i is given by:

$$\mathbb{E}[Z_i] = \sum_{m=0}^{\infty} B_i v^{m+1}\, {}_m p_x q_{x+m}\, t_{0i}^{(m)}. \tag{13}$$

Let random variable Z give the present value of the death benefit. Since there are n states, then:

$$Z = \sum_{i=0}^{n-1} Z_i. \tag{14}$$

The actuarial present value of the death benefit if the insured dies at any state is given by:

$$\mathbb{E}[Z] = \mathbb{E}\left[\sum_{i=0}^{n-1} Z_i\right] = \sum_{i=0}^{n-1} \sum_{m=0}^{\infty} B_i v^{m+1}\, {}_m p_x q_{x+m} t_{0i}^{(m)}. \tag{15}$$

Actuarial Present Value of Health Benefits

Let $H_i, i = 0,1,\dots,n-1$, where H_0 is the original face amount of the health insurance, represent the health benefit the insured can claim for the year at state i. The design of this insurance allows for the reduction or increase in the maximum health benefit, depending on the experience of the health insurance during the previous year. Now, there are different forms of health insurance: there are those that provide a fixed allowance for every day the insured is sick or in the hospital, and there are health insurances that pay for the medical expenses, which includes consultation fees, laboratory fees, *etc.*, in partial or in full such as those offered by health maintenance organizations.

Health Insurance Providing Fixed Hospitalization Income

For the first form, consider the hospitalization income insurance, which gives a daily income whenever the insured is admitted to a hospital as long as the policy is in force. The insured transitions from one state to another depending on the number of days he spends in the hospital in one policy year. Suppose the hospitalization income/benefit is paid at the end of the policy year.

Denote by N_m the number of days the insured spends in the hospital in the year m. Thus, $\mathbb{E}[N_m]$ is the expected number of days that the insured spends in the hospital in a year.

Let M_i^H give the present value of the hospitalization income/benefit at state i. The present value of hospitalization benefit is given by:

$$M_i^H = H_i N_m v^{m+1} \tag{16}$$

if the insured survives to age $x+m$ and is at state i during that year.

Again, assuming that the future lifetime and distribution of claims are independent, the probability that (x) survives to at least age $x+m$ while in state i is ${}_{m}p_{x}t_{0i}^{(m)}$. Thus, similar to Equation 7, the actuarial present value of the hospitalization income insurance at state i is:

$$\mathbb{E}[M_i^H] = \sum_{m=0}^{\infty} H_i\, \mathbb{E}[N_m]\, v^{m+1}\, {}_{m}p_{x}t_{0i}^{(m)}\,. \tag{17}$$

Note that we use v^{m+1} instead of v^m since the benefit is paid at the end of the policy year.

Let the random variable M^H give the present value of the total amount of hospitalization income/benefits the insured claimed while the policy is in force. Since there are n states, then:

$$M^H = \sum_{i=0}^{n-1} M_i^H. \tag{18}$$

The actuarial present value of the hospitalization income/benefit is given by:

$$\mathbb{E}[M^H] = \mathbb{E}\left[\sum_{i=0}^{n-1} M_i^H\right] = \sum_{i=0}^{n-1}\sum_{m=0}^{\infty} H_i\, \mathbb{E}[N_m]\, v^{m+1}\, {}_{m}p_{x}t_{0i}^{(m)}. \tag{19}$$

Health Insurance that Pays Medical Expenses

Now for the medical insurance, consider providing for the medical expenses of not more than a specific amount as long as the policy is in force. The insured transitions from one state to another depending on the medical expense spent in one policy year. Suppose again that the health benefit is paid at the end of the year.

Denote by C_m the medical expense at year m. If the insured is at state i at year m, then he/she will receive either C_m or H_i worth of medical benefits at the end of that year, whichever is lower.

Let M_i^E give the present value of the health benefit for the medical insurance if the insured is at state i. If the insured survives to age $x+m$ and is at state i during that year, then:

$$M_i^E = \min(H_i, C_m) v^{m+1}. \tag{20}$$

Thus, similar to Equation 17, the actuarial present value of the medical insurance at state i is:

$$\mathbb{E}[M_i^E] = \sum_{m=0}^{\infty} \min(H_i, \mathbb{E}[C_m])\ v^{m+1}\, {}_{m}p_{x}t_{0i}^{(m)}. \tag{21}$$

Let random variable M^E give the present value of the total amount of benefits claimed from the medical insurance while the policy is in force. Since there are n states, then:

$$M^E = \sum_{i=0}^{n-1} M_i^E. \tag{22}$$

The actuarial present value of the medical insurance benefits is:

$$\mathbb{E}[M^E] = \mathbb{E}\left[\sum_{i=0}^{n-1} M_i^E\right] = \sum_{i=0}^{n-1}\sum_{m=0}^{\infty} \min(H_i, \mathbb{E}[C_m])\ v^{m+1}\, {}_{m}p_{x}t_{0i}^{(m)}. \tag{23}$$

The Premium for a Combined Life and Health Insurance

Now, after deriving the actuarial present values of both life insurance and health insurance, defining the level annual premium is straightforward. If P is the level annual premium using the presented mechanism for the bonus-malus system, then the premium is:

$$P = \frac{\mathbb{E}[Z] + \mathbb{E}[M]}{\ddot{a}_x} \tag{24}$$

where $\mathbb{E}[M]$ is equal to either $\mathbb{E}[M^H]$ or $\mathbb{E}[M^E]$, depending on the type of health insurance.

ILLUSTRATIONS

Illustrations for the two forms of combining a life insurance product and a health insurance product are presented in this section, with the combined life and hospitalization income insurance preceding the combined life and medical insurance.

Here are the following benefits for each state:

$$B_i = \frac{B_0}{n}(n-i) \tag{25}$$

and:

$$H_i = \frac{H_0}{n}(n-i) \tag{26}$$

where B_0 and H_0 are the initial life benefit and initial health benefit, respectively. Essentially, all insured starts at state 0, which can be thought of as the best state. In state 0, the insured enjoys the life and health insurance benefit they chose upon purchase. Each state indicates the risk classification of an individual based on the previous year's claims. Individuals who always have good years will remain in state 0. However, individuals who experience bad years go up the state that corresponds to the claims they incurred and suffer from lower benefits. These individuals will be able to go down a state again if the claims they incurred are lower than their previous claims. These can be best described by the transition rule presented for each insurance.

Combined Life and Hospitalization Income Insurance

Recall that in this insurance, the insured transitions from one state to another depending on the number of days he spends in the hospital in one policy year denoted as N_m. Assume that N_m are independent and identically Poisson distributed.

Define the transition rule, $S_{i\to j}$, for transferring from state i to state j, from time m to $m+1$ as follows:

$$S_{i\to j} = \begin{cases} i-1, & N_m < i \\ i, & N_m = i \\ j, & N_m = j \text{ where } i < j \le n-1 \\ n-1, & N_m > n-1 \end{cases}. \tag{27}$$

Based on this transition rule, for $i = 0,1,2,\dots,n-1$: $t_{i,i-1} = \mathbb{P}(N_m < i)$ when $i \neq 0$, this is the probability that the insured goes down a state since the number of days spent in the hospital in the present year is less than the number of days spent in the hospital in the previous year; $t_{ii} = \mathbb{P}(N_m = i)$, this is the probability that the insured stays in the same state as the number of days spent in the hospital in the present year is the same as that in the previous year; $t_{ij} = \mathbb{P}(N_m = j)$, where $i < j \le n-1$, this is the probability that the insured goes up to state j since the insured spent j

days in the hospital in the present year, which is higher than that in the previous year; and $t_{i,n-1} = \mathbb{P}(N_m > n-1)$, this is the probability that the insured makes too many claims and is penalized by going up to the highest state.

Thus, the one-step probability of transition from state i to state j, for $i,j = 0,1,2,\dots,n-1$, is:

$$T = \begin{pmatrix} e^{-\lambda} & \lambda e^{-\lambda} & \frac{\lambda^2 e^{-\lambda}}{2} & \cdots & \frac{\lambda^{n-2}e^{-\lambda}}{(n-2)!} & 1-\sum_{k=0}^{n-2}\frac{\lambda^k e^{-\lambda}}{k!} \\ e^{-\lambda} & \lambda e^{-\lambda} & \frac{\lambda^2 e^{-\lambda}}{2} & \cdots & \frac{\lambda^{n-2}e^{-\lambda}}{(n-2)!} & 1-\sum_{k=0}^{n-2}\frac{\lambda^k e^{-\lambda}}{k!} \\ 0 & \sum_{k=0}^{1}\frac{\lambda^k e^{-\lambda}}{k!} & \frac{\lambda^2 e^{-\lambda}}{2} & \cdots & \frac{\lambda^{n-2}e^{-\lambda}}{(n-2)!} & 1-\sum_{k=0}^{n-2}\frac{\lambda^k e^{-\lambda}}{k!} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{\lambda^{n-2}e^{-\lambda}}{(n-2)!} & 1-\sum_{k=0}^{n-2}\frac{\lambda^k e^{-\lambda}}{k!} \\ 0 & 0 & 0 & \cdots & \sum_{k=0}^{n-2}\frac{\lambda^{n-2}e^{-\lambda}}{(n-2)!} & 1-\sum_{k=0}^{n-2}\frac{\lambda^k e^{-\lambda}}{k!} \end{pmatrix}.$$

Equations 15, 19, and 24 can now be used to price the premium. We further illustrate this result with specific parameters. We assume the following: age of the life insured is 20, with age at death assumed to be from 20–130 yr old; initial daily hospitalization benefit is PHP 10,000 (H_0); interest rate is 5% ($\hat{\imath}$); the maximum death benefit is PHP 1,000,000 (B_0); and the number of states is 10. The mortality assumption considered is the standard ultimate survival model. We compute for the premium price with varying values of the Poisson parameter λ from 1–20. Note that λ is the mean of number of claims, meaning we consider here an insured who spends around 1–20 d in the hospital in a year. Figure 1 shows the premium price plotted against λ. It can be seen that the premium price increases as λ increases, *i.e.* the premium increases as the expected number of days in the hospital increases.

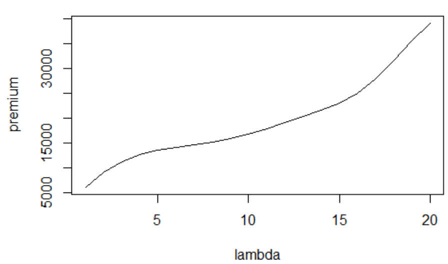

Figure 1. Premium price for combined life and hospitalization insurance.

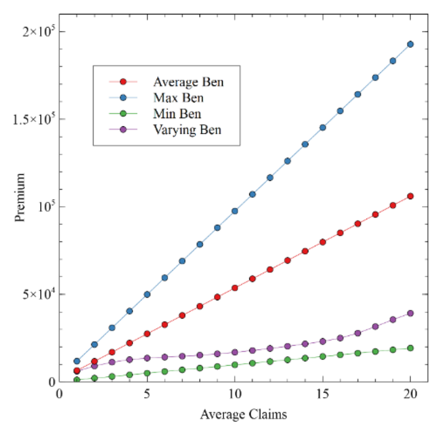

Figure 2. Premium prices comparison of combined life and hospitalization insurance with varying benefits and fixed benefits.

Figure 2 shows the premiums derived when using the presented bonus-malus system with varying benefits compared to those with fixed death and hospitalization income benefits. The fixed benefits used are the maximum benefits (B_0 and H_0), minimum benefits (B_n and H_n), and the average of the benefits. It is observed that the premium prices derived are less compared to those with fixed maximum and average benefits and are close to the premium prices derived when using fixed minimum benefits. Notice that when the risk is high, *i.e.* when λ is high, the premium price is still less which can benefit the insured lives with low risk. Also, the insurer is also protected from a huge number of claims due to the mechanism of varying benefits. An advantage of the proposed system can also be observed since the insured pays a small premium price but can receive benefits as high as the maximum benefits.

Combined Life and Medical Insurance

Let each C_m, the medical expense at year m, be independent and identically gamma-distributed. Let $L = H_0$ be the medical expense threshold set for monitoring the insured. Define the transition rules, $S_{i\to j}$, by:

$$S_{i\to j} = \begin{cases} i-1, & C_m \in \left[0, \frac{L}{n} i\right) \\ i, & C_m \in \left[\frac{L}{n} i, \frac{L}{n}(i+1)\right) \\ j, & C_m \in \left[\frac{L}{n} j, \frac{L}{n}(j+1)\right) \text{ where } i < j \le n-1 \\ n-1, & C_m \in [L, \infty) \end{cases} . \qquad (28)$$

Based on this transition rule, for $i = 0,1,2,\dots,n-1$, $t_{i,i-1} = \mathbb{P}\left(0 \le C_m < \frac{L}{n} i\right)$ when $i \ne 0$, $t_{ii} = \mathbb{P}\left(\frac{L}{n} i \le C_m < \frac{L}{n}(i+1)\right)$, $t_{ij} = \mathbb{P}\left(\frac{L}{n} j \le C_m < \frac{L}{n}(j+1)\right)$ where $i < j \le n-1$, and $t_{i,n-1} = \mathbb{P}(C_m \ge L)$. In essence, the claim in the present year dictates the state for the succeeding year. If the claim in the present year is reasonably less than the previous year, the insured goes down one state. If it is almost the same as last year, the insured stays in the same state. If it is reasonably higher, then the insured goes up a state. However, if it is significantly higher, then the insured goes to the highest state.

Thus, the one-step probability of transition from state i to j is:

$$T = \begin{pmatrix}
\int_0^{\frac{L}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{L}{n}}^{\frac{2L}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{2L}{n}}^{\frac{3L}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \cdots & \int_{\frac{L(n-2)}{n}}^{\frac{L(n-1)}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{L(n-1)}{n}}^{\infty} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx \\
\int_0^{\frac{L}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{L}{n}}^{\frac{2L}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{2L}{n}}^{\frac{3L}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \cdots & \int_{\frac{L(n-2)}{n}}^{\frac{L(n-1)}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{L(n-1)}{n}}^{\infty} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx \\
0 & \int_0^{\frac{2L}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{2L}{n}}^{\frac{3L}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \cdots & \int_{\frac{L(n-2)}{n}}^{\frac{L(n-1)}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{L(n-1)}{n}}^{\infty} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \int_{\frac{L(n-2)}{n}}^{\frac{L(n-1)}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{L(n-1)}{n}}^{\infty} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx \\
0 & 0 & 0 & \cdots & \int_0^{\frac{L(n-1)}{n}} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx & \int_{\frac{L(n-1)}{n}}^{\infty} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}dx
\end{pmatrix}.$$

Equations 15, 23, and 24 can now be used to price the premium. Once more, giving specific values to illustrate the model much better and using the values given in the previous section but changing the health benefit to initial maximum medical benefit of PHP 200,000 (H_0) is presented here. With gamma parameters $\alpha,\beta \in [1,50]$ and using adjusted values (in thousand PHP), the range of mean values of the distribution $\alpha\beta$ are from 1–2,500 (in thousand PHP). These values are expected to cover a wide range of possible claim distributions of the insured but note that the insurance will only cover up to the maximum health benefit, which starts at H_0= PHP 200,000. Figure 3 shows the plot of the premium price against α and β. What is interesting here is that after some combination of α and β, the computed premium seems to plateau. This can be seen as an advantage to high-risk individuals who are looking for insurance coverage as the premium does not seem to raise so much even if their mean claims $\alpha\beta$ are large.

Figure 4 shows the premiums derived when using the presented bonus-malus system with varying benefits compared to those with fixed death and hospitalization benefits. The scale parameter β is fixed at 5, while the shape parameter α has values from 1–50. This gives us an idea regarding the behavior of the premium price, given varying skewness of the claim distributions and mean values ranging from 5–250 (in thousand PHP). The horizontal axis represents the mean of a gamma distribution. It is observed that the premium prices derived using the presented system are less compared to those with fixed maximum and average benefits and are higher than the premium prices derived when using fixed minimum benefits. Again, notice that when the risk is high (the mean is high), the premium price is still less which can benefit the insured lives with low risk, and the insurer is also protected from big amounts of claims due to the mechanism of varying benefits. A similar scenario can also be observed when considering other values for β.

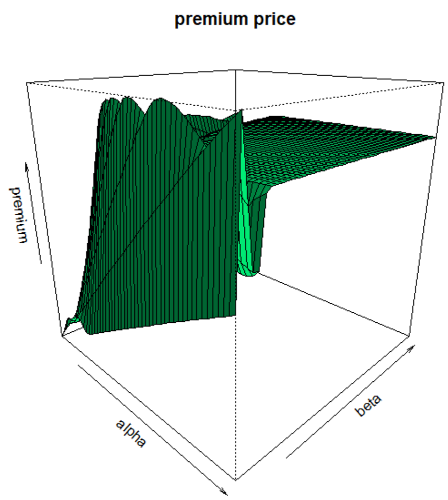

Figure 3. Premium price for combined life and medical insurance.

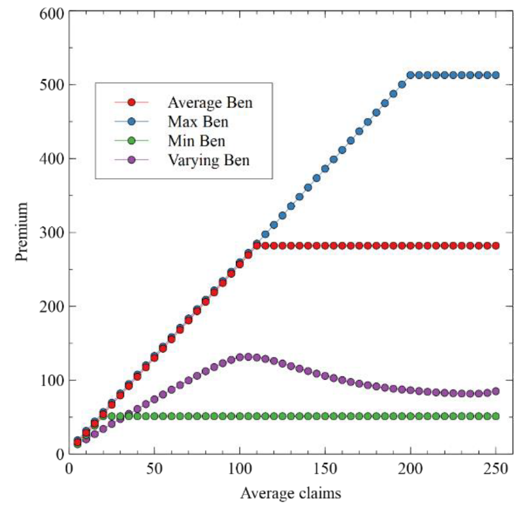

Figure 4. Premium prices comparison of combined life and medical insurance with varying benefits and fixed benefits.

CONCLUSION

This study gives a framework for combining life and health insurance while assuring that the actual claims experience of the life insured is well-reflected to the premiums. A general model for the premium price is given as well as two illustrations: one combining a life and hospitalization income insurance and another combining a life and medical insurance. More precise models of the premium are also derived, and numerical illustrations are given for both situations. Numerical illustrations show the advantage of the proposed mechanism since the insured pays a small premium price but can still receive high benefits.

While no empirical evidence can be presented in this case, this study claims that adverse selection can be reduced by employing this system of combining life and health insurance. Intuitively, the level premium will solve the dropping out of the low-risk people caused by high premium. This dropping out is known to cause insurance failure (Cummins *et al.* 1983; Di Novi 2011). The welfare of the low-risk people is also better in this insurance than in regular insurance since they do not suffer an increase in premium caused by the claims of the high-risk people, and they also receive incentives in the form of an increase in the benefit. These, while not directly tackle how adverse selection can be reduced, constitute a step towards that direction.

Future studies can be made by relaxing the assumption of independence of the probability of death and the probability of health claims. This is a crucial assumption of this study, and the authors hypothesize that a different model would be derived by considering the dependence of the probability of health claims on the probability of death or *vice versa*. Another study that can be made is an empirical study or at least a numerical simulation or studying the price elasticity of demand to prove that this framework indeed reduces adverse selection.

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