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Design of Robust Optimal Fractional-Order PID Controllers Using Particle Swarm Optimization Algorithm for Automatic Voltage Regulator (AVR) System

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In this paper, a fractional order controller with a first-order low pass filter in derivative was designed. Since disturbance rejection is more common than set point tracking in industrial processes, the performance of the system was evaluated by its ability to reject disturbance. Morever, a method for robust optimum tuning of fractional PID controllers for AVR system using Particle swarm optimization (PSO) algorithm was presented. Using the H_∞-norm of a SISO linear system, condition for disturbance rejection was determined and constrained optimization problem was solved. The proposed approach with new defined fitness function has very easy implementation and has the most control performance. The influence and efficiency of the proposed method were illustrated in simulations.

Key words: AVR system; fitness function; H_∞-norm; Robust optimal fractional-order PID controller design

INTRODUCTION

In the last decade, fractional-order dynamic systems and controllers had been widely studied in many areas of engineering and science (Baleanu et al. 2012a; Baleanu et al. 2012b; Pan and Das 2013). The concept of fractional-order PID (FOPID) controllers was proposed by Podlubny in 1999. Hardware and digital realizations of fractional-order systems can be followed in Valerio and Sada (2011). Biswas et al. (2009) presented an FOPID design method based on differential evolution (DE) technique. Moreover, Yeroglu and Tan (2011) presented a method based on the Ziegler–Nichols and the Astrom–Hagglund methods. Luo et al. (2011a) also designed a fractional order $(PI)^\lambda$ controller to improve the flight control performance of a small fixed-wing unmanned aerial vehicle (UAV) . Barbosa et al. (2010) considered the effect of fractional orders in the velocity control of a servo system. Luo and Chen (2009) focused on a given type of simple model of fractional order system and proposed a fractional order [proportional derivative] (FO-[PD]) controller for this class of fractional order system. An experimental study of the fractional order proportional and derivative (FO-PD) controller for the fractional order systems with generalized fractional capacitor membrane model was presented by Luo et al. (2011b) to validate the control performance.

Optimal tuning of classical PID controller parameters was done in other literature but with proposed fitness functions and classical PID which cannot achieve a high-quality solution that effectively improves the transient response of the controlled system. Devaraj and Selvabala (2009) presented a method based on real-coded genetic algorithm and fuzzy logic. An optimal design method for determining the PID controller parameters of the AVR system using the

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PSO algorithm had been proposed in many studies (Amer et al. 2008; Gaing 2004; Rahimian & Raahemifar 2011). Other methods such as genetic algorithm (GA), simulated annealing (SA), bee colony algorithm, and chaotic algorithm have been used for achieving high efficiency and searching global optimal solution in problem space (Coelho 2009; Gozde and Taplamacioglu 2010; Krohling and Rey 2001; Wong et al. 2009). Zamani et al. (2009) presented a fractional order controller for AVR system based on a new criterion function with eight terms by using particle swarm optimization. Domingues et al. (2009) introduced a fractional order PID controller for AVR system. An optimum fractional order PID controller was presented by Tang et al. (2012) using chaotic ant swarm (CAS) optimization method and the same fitness function used by Gaing (2004). Bingul and Karahan (2011) tuned fractional PID controllers using PSO algorithm for robot trajectory control by applying Mean of Root of Squared Error (MRSE), Mean of Absolute Magnitude of the Error (MAE), and Mean Minimum Fuel and Absolute Error (MMFAE) as fitness functions. In earlier study, Das and Pan (2013) presented a fractional order controller for AVR system based on chaotic multi-objective optimization using frequency domain techniques. Das and Pan (2012) presented a fractional order PID controller using Chaotic multi-objective optimization.

In this paper, a practical optimal FOPID controller with disturbance rejection ability in an AVR system was designed. Parameters of robust FOPID controller were determined through optimizing a new proposed cost function in order to achieve disturbance rejection.

METHODOLOGY

Review on fractional calculus

Fractional-order PID controllers (FOPID)

The fractional PID controller is a generalization of the PID controller. The transfer function of this controller is given by the following function:

$$C(s) = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu} \qquad (1)$$

In practical applications, the pure derivative action is never used due to the "derivative kick" produced in the control signal for a step input and to the undesirable noise amplification. It is usually replaced by a first-order low pass filter; thus, the Laplace transformation of the fractional PID controller can be written as:

$$C(s, K) = k_p \left(1 + \frac{1}{T_i s^{\lambda}} + \frac{T_d s^{\mu}}{1 + \frac{T_d}{N} S} \right) \qquad (2)$$

where $K = [k_p, T_i, \lambda, T_d, \mu]^T$, k_p is the proportional constant, T_i is the integration constant, T_d is the differentiation constant, and λ and μ are positive real numbers.

Grünwald-Letnikov (GL), Riemann-Liouville (RL), and Caputo definitions have been known as important definitions of fractional derivatives or integrals in fractional calculus. Definition of the Grünwald-Letnikov fractional-order derivative is given by the following equation: (Caponetto et al. 2010)

$${}_aD_t^{\gamma} f(t) = \lim_{h \to 0} h^{-\gamma} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\gamma}{j} f(t - jh) \qquad (3)$$

where $(-1)^j \binom{r}{j}$ are (Das 2011):

$$c_0^{(r)} = 1,\ c_j^{(r)} = \left(1 - \frac{1+r}{j} \right) c_{j-1}^{(r)}, \qquad j = 0,1,2,\ldots \qquad (4)$$

and used for recursive computation, these are weights.

Definition of the Reimann-Liouville (RL) fractional-order derivative is given by:

$${}_aD_t^{\gamma} f(t) = \frac{1}{\Gamma(n-\gamma)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\gamma-n+1}} d\tau \qquad (5)$$

where $\Gamma(.)$ is Euler's Gamma function that generalizes the factorial, and allows operator, to take non-integer values.

Caputo definition is given by:

$${}_a^C D_t^{\gamma} f(t) = \frac{1}{\Gamma(n-\gamma)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\gamma-n+1}} d\tau \qquad (6)$$

More detail is available in Das (2011).

Oustaloup approximation algorithm

Oustaloup's approximation method uses a band-pass filter to approximate the fractional – order operator s^{λ} based on frequency – domain response (Das 2011). Oustaloup approximation of a continuous fractional order operator s^{λ} is as follows:

$$G_f(s) = A \prod_{n=-M}^{M} \frac{s + \omega_n'}{s + \omega_n} \qquad (7)$$

where the zeros, poles and the gain can be obtained, respectively, as:

$$\omega_n' = \omega_b \left(\frac{\omega_h}{\omega_b} \right)^{\frac{n+M+\frac{1}{2}(1-\lambda)}{2M+1}} \qquad (8)$$

$$\omega_n = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{n+M+\frac{1}{2}(1+\lambda)}{2M+1}} \quad (9)$$

$$A = \left(\frac{\omega_h}{\omega_b}\right)^{-\frac{\gamma}{2}} \prod_{n=-M}^{M} \frac{\omega_n}{\omega'_n}. \quad (10)$$

In simulation, for approximation of s^{λ} frequency range is closed as: $\omega\in[\omega_b,\omega_h]$ and $\omega_b = 0.001$, $\omega_h=1000$, $M=2$.

Problem description

The single-input single-output feedback control system with an external disturbance d(t) and $PI^{\lambda}D^{\mu}$ controller is shown in Figure 1. It is assumed that the disturbance has a step or sinusoidal function known form. W(s) is a weighting function consisting of a low-pass filter such as to reject the frequency response of the external disturbance d(t). Here, G(s) and S(s) are the process and sensor, respectively.

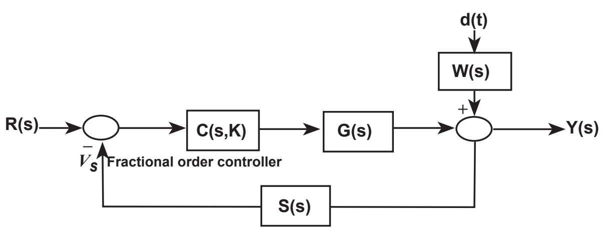

Figure 1. control system with disturbance.

Sufficient condition for disturbance rejection

Theorem: For Figure 1, system C(s, k) is a robust optimal fractional controller if

$$\underset{K}{Min}J(K) = e^{-\beta}(T_s + T_r) + (1 - e^{-\beta})\,(ITSE + M_p) \quad (11)$$

subject to $\|\alpha(s,K)\|_{\infty} < \varepsilon$

where $\varepsilon < 1$, $K = [k_p,\ T_i,\ \lambda\ k_d,\ \mu]^T$, T_s settling time, T_r rise time, M_p overshoot, β is weighting factor and $ITSE$ is integral of the time multiplied square error criterion given by:

$$ITSE = \int_0^{t_{sim}} te^2(t)\ dt \quad (12)$$

where t_{sim} is total simulation time and $e(t) = R(t) - V_S$ is the tracking error.

The PID controller design method using the integral of absolute error (IAE), integral of square error (ISE), or integral of time multiplied square error (ITSE) is often employed in control system design. The IAE, ISE performance indices are as follows:

$$IAE = \int_0^{\infty} |e(t)|\,dt \quad (13)$$

$$ISE = \int_0^{\infty} e^2(t)\ dt \quad (14)$$

Proof: If R(s) =0, then, the disturbance rejection constraint can be described as:

$$\|\alpha(s,K)\|_{\infty} < \varepsilon \quad (15)$$

where $\alpha(s, K) = \frac{Y(s)}{D(s)} = \frac{W(s)}{1 + C(s,K)G(s)(S(s)}$,

$K = [k_p,\ T_i,\ \lambda\ k_d,\ \mu]^T$, $\varepsilon < 1$ is the desired rejection level, and $\|.\|_{\infty}$ denotes the H_{∞}-norm. The H_{∞} -norm of a SISO linear system is the peak gain of the frequency response .Thus, for a continuous-time system H(s), the H_{∞} -norm is given by:

$$\|H(s)\|_{\infty} = \max\ |H(j\omega)| \quad (16)$$

The *ITSE* performance index has excellences of smaller overshoot and oscillation than the IAE or the ISE performance indices. It is also the most sensitive and has the best selectivity.

Design of $PI^{\gamma}D^{\mu}$ PSO-robust controller

Particle Swarm Optimization (PSO)

The PSO algorithm was developed in 1995 (Kennedy and Eberhart 1995). PSO is a robust stochastic optimization technique based on the movement and intelligence of swarm. It applies the concept of social interaction to problem solving.

In PSO algorithm, the *Jth* particle (k_j) is treated as a point in an N-dimensional space ($k_j = (k_{j,1}, k_{j,2}, \ldots, k_{j,N})$ which adjusts its "flying" according to its own flying experience as well as the flying experience of other particles. Each particle keeps track of its coordinates in the solution space which are associated with the best solution that has been achieved so far by that particle. This value is called personal best, *Pbest*. The previous position of the jth particle in an N-dimension space is given by:

$$pbest_j = (pbest_{j,1}, pbest_{j,2}, \ldots, pbest_{j,N})$$

Another best value that is tracked by the PSO is the best value obtained so far by any particle in the neighborhood of that particle. This value is called *gbest*.

The basic concept of PSO lies in accelerating each particle toward its *pbest* and the *gbest* locations, with a random weighted acceleration at each time step. The modification of the particle's position can be mathematically modeled according to the following equations:

$$v_{j,N}^{(t+1)} = \omega.v_{j,N}^{(t)} + c_1 rand_1(\ldots)\left(pbest_{j,N} - K_{j,N}^{(t)}\right) + c_2 rand_2(\ldots)\left(gbest - K_{j,N}^{(t)}\right) \quad (17)$$

$$K_{j,N}^{(t+1)} = K_{j,N}^{(t)} + v_{j,N}^{(t+1)} \quad (18)$$

The following constraints for velocity in each iteration is applied as:

$$v_{j,N}^{(t+1)} = \begin{cases} V_N^{\max} & if \ v_{j,N}^{(t+1)} > V_N^{\max} \\ -V_N^{\max} & if \ v_{j,N}^{(t+1)} < -V_N^{\max} \end{cases} \tag{19}$$

where $V_N^{\max}$ is the maximum possible magnitude of velocity of any particle in the Nth dimension, j =1,2,…,n, N = 1,2,…, m.

A description of the parameters of equations (17), (18), and (19) is given in Table 1.

Changing velocity by this way enables the Jth particle to search around its local best position, pbest, and global best position, gbest. In many experiences with PSO, $V_N^{\max}$ is often set to the maximum dynamic range of the variables on each dimension. Suitable selection of weighting function ω in Equation (10) provides a balance between the global and local explorations, thus requiring less iteration, on the average, to find a sufficiently optimal solution. ω often decreases linearly from ($\omega_{\max}$) about 0.9 to ($\omega_{\min}$) about 0.4 during a run. Weighting function ω is set by:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} \times iter \tag{20}$$

where $iter_{\max}$ is the maximum number of iterations and iter is the current number of iterations.

The advantages of PSO over other traditional optimization techniques have been stated in Cvetkovski and Petkovska (2013) and in the references therein.

Proposed fitness function

The most difficult step in applying PSO is to choose the best fitness function which is used to evaluate the fitness of each particle. The proposed method is described in the form of an algorithm in Figure 2 and is further explained as follows:

[**Step 1**] Initialize particles with random position and velocity vectors.

[**Step 2**] For each particle's position, calculate $\|\alpha(s,K)\|_\infty$ in the worst state $\|\alpha(s,K)\|_\infty > \varepsilon$ then select $\|\alpha(s,K)\|_\infty$ as fitness function and continue PSO algorithm for its minimization.

[**Step 3**] As soon as $\|\alpha(s,K)\|_\infty < \varepsilon$, select $e^{-\beta}(T_s + T_r) + (1 - e^{-\beta})\,(ITSE + M_p)$ as new fitness function and continue PSO algorithm for its minimization.

Everywhere $\|\alpha(s,K)\|_\infty > \varepsilon$ refer to [Step 2]. Thus, fitness function is defined as follows:

$$\underset{K}{Min}\, J(K) = \begin{cases} e^{-\beta}(T_s + T_r) + (1 - e^{-\beta})(ITSE + M_p), & \|\alpha(s,K)\|_\infty < \varepsilon \\ \|\alpha(s,K)\|_\infty & , \ \|\alpha(s,K)\|_\infty > \varepsilon \end{cases} \tag{21}$$

The most important parameter in this fitness function is ITSE. Minimization of this parameter forces t_s, t_r and M_P parameters to be optimum. In the other word, with this fitness function t_s, t_r and M_P parameters are optimized directly and indirectly.

Remark 1. In Equation (21), weighted cost function and sensitivity function was not used because selecting appropriated weighted factor is not convenient. Using the proposed method, the best tradeoff between robustness

Table 1. Description of Particle Swarm Optimization (PSO) parameters.

Parameter	Description
n	number of particles in the population (population size)
m	dimension of problem (number of members in a particle) that there is five
t	pointer of iterations (generations)
$v_{j,N}^{(t)}$	Velocity of particle j at iteration t, $V_N^{min} \le v_{j,N}^{(t)} \le V_N^{max}$
ω	weighting function
C_1, C_2	acceleration factors
$rand_1(\ldots)$, $rand_1(\ldots)$	uniformly distributed random numbers between 0 and 1
$K_{j,N}^{(t)}$	Current position of particle j at iteration t
$pbest_{j,N}$	pbest position of particle j
$pbest$	best position of swarm

Table 2. Components of AVR system model with transfer function and parameters limits.

	Transfer function	Parameters limits
Fractional order controller	$kp\,(1+\frac{1}{T_i s^{\lambda}}+\frac{T_d s^{\mu}}{1+\frac{T_d}{N}s})$	$0 \le k_p \le 10$, $N = 64$ $0 \le T_i \le 20000$, $0 \le T_d \le 1$ $0 \le \lambda \le 2$; $0 \le \mu \le 2$
Amplifier	$TF_{amplifier} = \frac{K_a}{1+\tau_a s}$	$10 \le K_a \le 40$; $0.02 \le \tau_a \le 0.1$
Exciter	$TF_{exciter} = \frac{K_s}{1+\tau_e s}$	$1 \le K_e \le 10$; $0.4 \le \tau_e \le 1$
Generator	$TF_{generator} = \frac{K_g}{1+\tau_g s}$	$0.7 \le K_g \le 1$; $1 \le \tau_g \le 2$
Sensor	$S(s) = \frac{K_s}{1+\tau_s s}$	$K_s = 1$; $0.001 \le \tau_s \le 0.06$

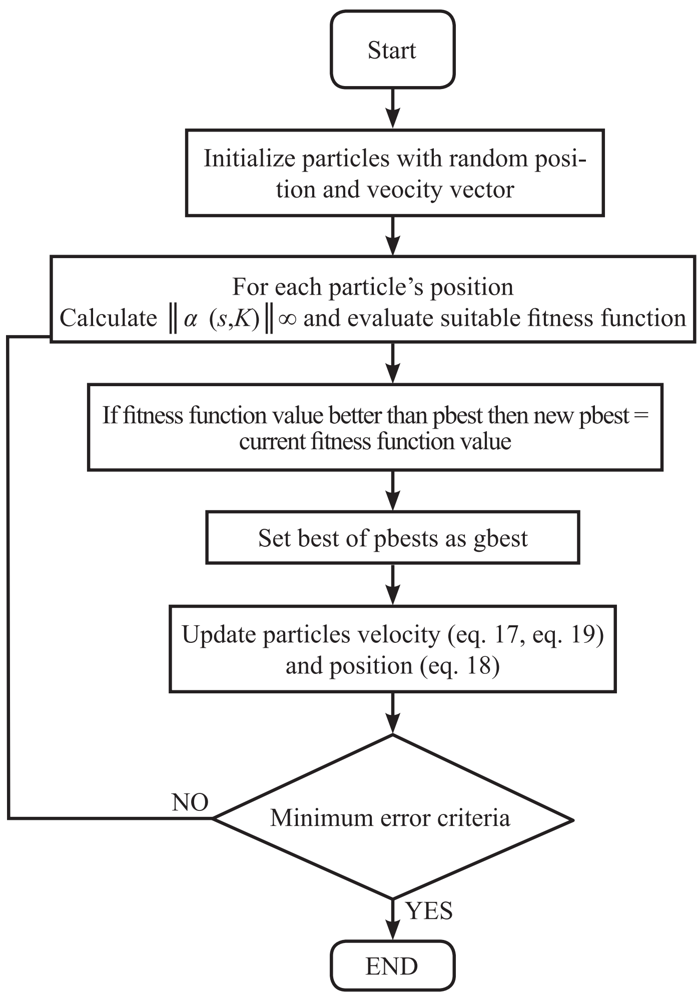

Figure 2. The Particle Swarm Optimization-Fractional Order PID robust controller flowchart.

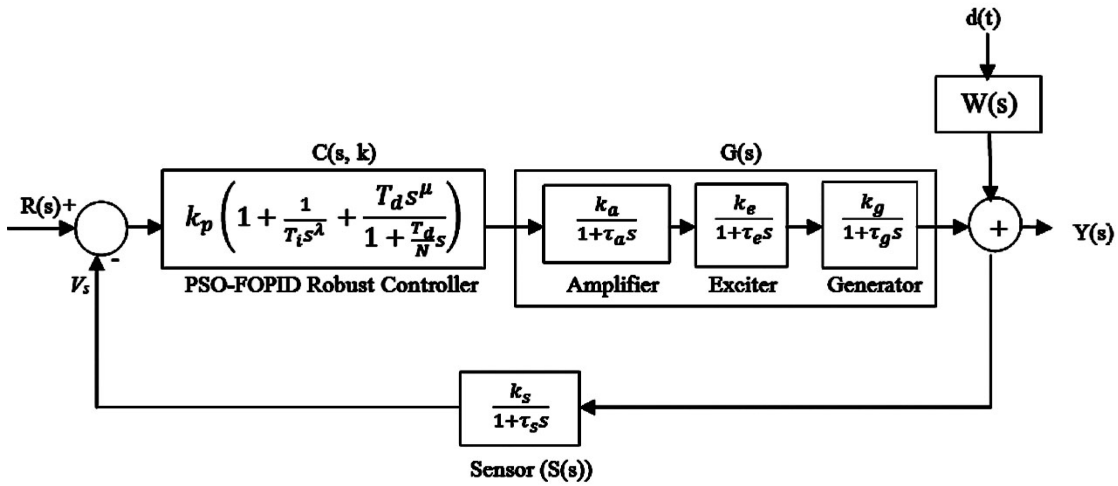

Figure 3(a). Block diagram of AVR system.

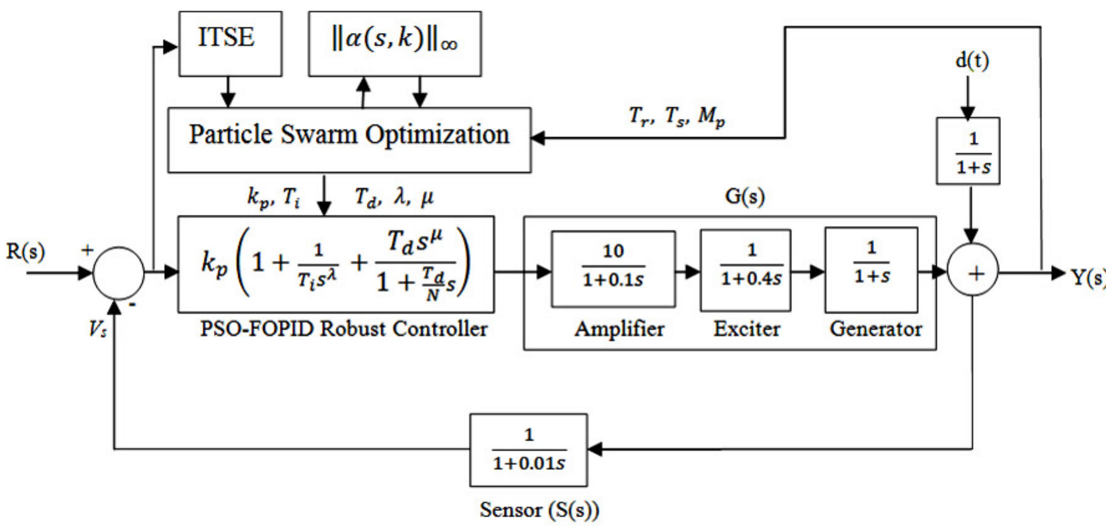

Figure 3(b). Proposed method implementation for AVR system with Particle Swarm Optimization-Fractional Order PID robust controller.

and the time domain performance is directly selected.

Modeling of AVR system

An AVR system holds the terminal voltage magnitude of a synchronous generator at a specified level. Therefore, the stability of the AVR system would seriously affect the security of the power system and the design of a controller for an AVR system is necessary to improve its stability and transient performance.

A simple AVR system is comprised of four main components: amplifier, exciter, generator and sensor. A small signal model of this system, including PSO- FOPID robust controller, is shown in Figure 3(a) and the limits of the parameters used in it are presented in Table 2.

Simulation results

A proposed method implementation for AVR system with weighting function $W(s)=\frac{1}{1+s}$ is illustrated in Figure 3(b). The external disturbance is considered to be $d(t) = 0.1\sin t$ or unit step and the disturbance attenuation level specified is $\varepsilon = 0.1$. PSO-FOPID robust controller is designed as:

$$C(s, K^{*}) = k_p\left(1+\frac{1}{T_i s^{\lambda}}+\frac{T_d s^{\mu}}{1+\frac{T_d}{N}s}\right) \tag{22}$$

where $K^* = [k_p, T_i, \lambda, T_d, \mu]^T = [8.3294, 16658.8, 1.9536, 0.0768, 1.3561]^T$, β and N are assumed to be 2.5 and 64, respectively.

RESULTS AND DISCUSSION

In this paper, PSO parameters are selected as: $c_1=c_2=2$, maxIter =100, population size=100, $\omega_{min} = 0.4$, and $\omega_{max} = 0.9$. The magnitude of the disturbance rejection constraint ($|\alpha(s,K^*)|$) for the optimal vector of controller parameters $K^* = [8.3294, 16658.8, 1.9536, 0.0768, 1.3561]^T$ is shown in Figure 4. The disturbance rejection norm value is $\|\alpha(s,K^*)\|_\infty=0.0867$. Because this value is smaller than $\varepsilon = 0.1$, the condition for disturbance rejection is satisfied. The step response of the control system with $C(s,K^*)$ PSO-FOPID robust controller for d(t)=0 and sinusoidal disturbance d(t)= 0.1 sin t is shown in Figure 5 with blue and green color respectively. Also, red color in Figure 5 shows the step response of AVR system without PSO-FOPID robust controller in presence of sinusoidal disturbance. Figure 6 shows the step response of the control system with $C(s,K^*)$ PSO-FOPID robust controller for d(t) = 0 and the unit step disturbance with blue and green color respectively. Also, red color in Figure 6 shows the step response of AVR system without PSO-FOPID robust controller in presence of unit step disturbance signal. Figures 5 and 6 present almost no difference compared to the without disturbance case. Therefore, the external disturbance on the plant output has little influence on the step response because the maximum value of the constraint for disturbance rejection $\|\alpha(s,K^*)\|_\infty$ is 0.0867 which is small. The convergence characteristic of the PSO-fractional PID robust controller is shown in Figure 7.

Table 3. Additional representative solutions for the PSO-fractional PID robust controller**.**

k_p	T_i	λ	T_d	μ
8.3294	16658.8000	1.9536	0.0768	1.3561
8.4318	16658.8091	1.9123	0.0751	1.3912
8.4407	16658.8056	1.9098	0.0762	1.4408
8.5775	16658.8143	1.9154	0.0711	1.46111
8.5379	16658.9088	1.8636	0.0844	1.4809
8.5435	16658.9155	1.8520	0.0889	1.4115
8.5115	16658.9453	1.8452	0.0876	1.5127
8.5599	16658.8742	1.8643	0.0846	1.5342
8.6049	16658.8923	1.9235	0.0860	1.5657
8.7339	16658.8183	1.8973	0.0869	1.5974
8.7863	16658.8550	1.8334	0.0945	1.6139
8.8111	16658.8259	1.8468	0.0977	1.6463
8.8655	16658.8237	1.8567	0.0995	1.6490
8.8655	16658.7389	1.8582	0.0999	1.6719
8.8668	16658.7358	1.8499	0.1042	1.6822
8.8658	16658.7046	1.8621	0.1150	1.6952
8.8526	16658.7273	1.8508	0.1247	1.7025
8.8291	16658.6882	1.8631	0.1296	1.7085
8.8889	16658.7057	1.8559	0.1293	1.7233
8.8815	16658.7185	1.8548	0.1379	1.7239
8.9152	16658.7290	1.8483	0.1401	1.7482
8.8935	16658.7061	1.8449	0.1404	1.7608
8.9121	16658.7140	1.8359	0.1436	1.7620
9.0563	16658.6938	1.8413	0.1461	1.7705
9.0487	16658.7445	1.9963	0.1507	1.7708
8.8627	16658.7146	1.8294	0.1526	1.7804
8.9071	16658.6745	1.8710	0.1535	1.7945
8.9124	16658.6709	1.8253	0.1574	1.8035
8.8930	16658.6820	1.8408	0.1651	1.7984
8.8973	16658.6507	1.8413	0.1720	1.7973

Table 4. Standard deviation and mean of the parameters of the PSO-fractional PID robust controller in Table 3.

Parameter	Standard deviation	Mean
k_p	0.1920	8.7694
T_i	0.0829	16659
λ	0.0393	1.8681
T_d	0.0314	0.1157
μ	0.1378	1.6452

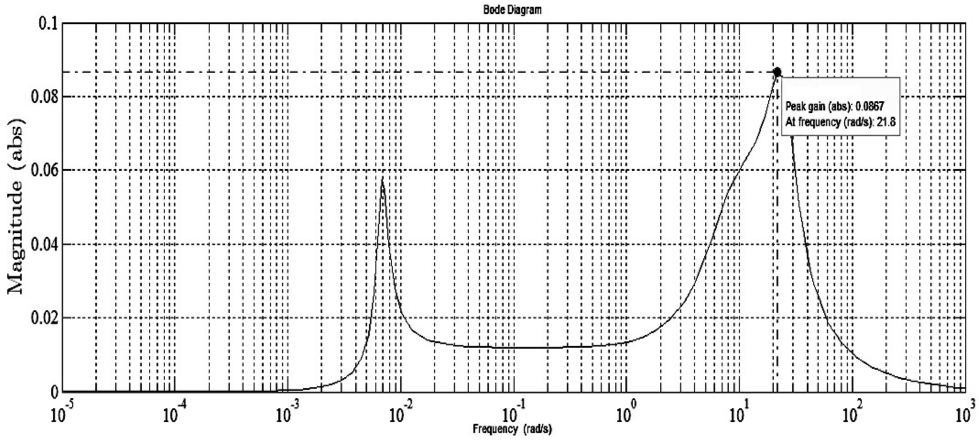

Figure 4. The magnitude of the disturbance rejection constraint $(|\alpha(s,K^*)|)$.

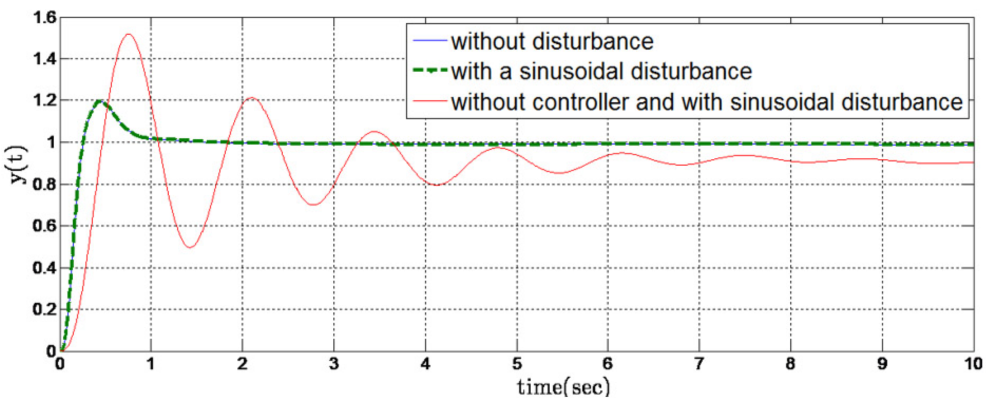

Figure 5. The step response of the control system with PSO-FOPID robust controller for d(t) = 0 and sinusoidal disturbance d(t) = 0.1 sin *t* and without controller in presence of sinusoidal disturbance.

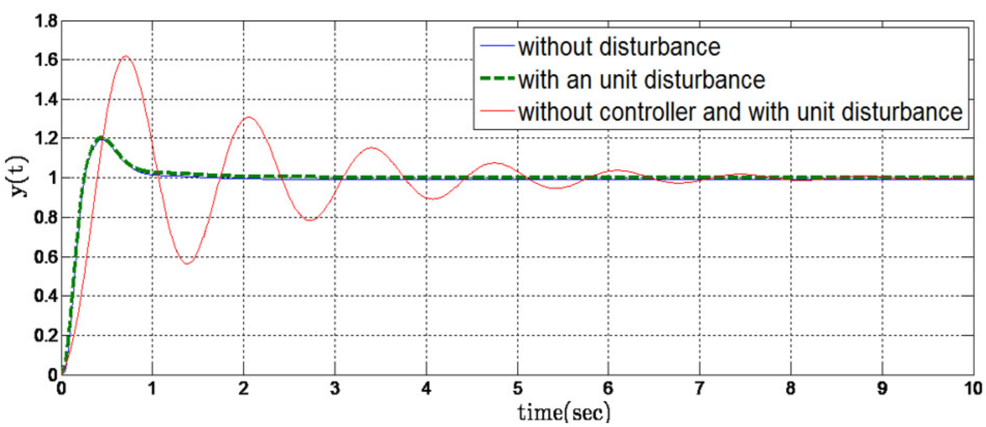

Figure 6. The step response of the control system with PSO-FOPID robust controller for d(t) = 0 and unit step disturbance and without controller in presence of unit disturbance.

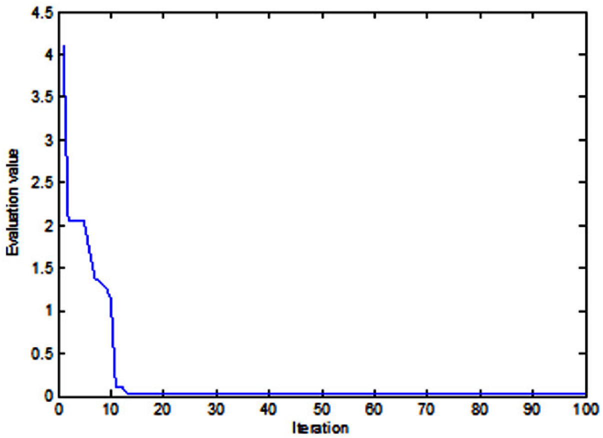

Figure7. The convergence characteristic of the PSO-fractional PID robust controller.

In this method, the effectiveness of the disturbance can be limited significantly. PSO algorithm was run 30 times and additional representative solutions for the PSO-fractional PID robust controller design was obtained and presented in Table 3. The mean and standard deviation values are reported in Table 4.

CONCLUSIONS

In this paper, a robust fractional order controller with a first-order low pass filter in derivative was designed and a method for robust optimum tuning of fractional order PID controllers for AVR system using PSO algorithm was presented. The simulation results illustrate that the external disturbance on the plant output has little influence on the step response. In this method, the effectiveness of the disturbance can be limited significantly. Since PID control signal is smooth, the proposed technique can be applied as an efficient method for the robust optimal design of fractional-order controllers in practical systems. Currently, the proposed method is being applied on a practical AVR system in an electrical machine laboratory in the Gonabad branch of Islamic Azad University using programmable logic control (PLC). The proposed method in this paper is clearly optimal and more robust for disturbance signal but the design of a robust optimal controller for model uncertainties in AVR systems is still an open problem in this field and thus can be considered in future research works.

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